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On the Physics of Harmonicas

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Several studies and researches carried out in connection with the manufacture of harmonicas are briefly reviewed. A stroboscopic device used in tuning the harmonica reeds and a constant pressure device for sounding them during test are described. The manufacture of the reeds is discussed in some detail. It is pointed out that the primary problem in the manufacture of harmonicas is to produce them at a cost which will meet European competition. It is concluded that harmonica manufacture presents interesting problems for the physicist, engineer, and metallurgist.

IN common with most other types of musical instruments the development of the harmonica has been an art rather than a science. Furthermore, the production of this instrument has been for many years practically a monopoly in the hands of a single German manufacturer (the M. Hohner Company). A number of attempts to produce harmonicas in quantity in the United States have not survived foreign competition. In connection with one of these attempts the author conducted, during the last war, a number of studies and researches, the results of which will be briefly reviewed here.[1]

1. METHOD OF TUNING

Initially it was thought that the primary difficulty encountered in the mass production of harmonicas would be the tuning. Parenthetically, in this connection it should be pointed out that it is not possible, even in the highly developed state of European manufacture, to produce reeds with exactly the correct pitch, (although this ideal can be closely approached) so that each reed (20 reeds, for example, in a 10-hole harmonica) must be tuned finally individually. This is done by filing slightly the upper surface of the reed near the fixed end (thus reducing the restoring force) if the pitch of the reed is too high, or near the free end (thus reducing the inertia) if the pitch is too low. Usually it is necessary merely to "touch up" the reed by drawing the edge of a small three-cornered file one or more times transversely across the reed. These file marks appear as tiny scratches when the reed is examined.

In Germany the tuning is carried out by ear and is said to be an art handed down largely from father to son in communities specializing in harmonica manufacture. Such personnel obviously would not be available immediately to a manufacturer in the United States, and the cost of training it would, moreover, be prohibitive. However, this difficulty turned out *not* to be the primary one, or if it was, it was easily sur-

[1] The work described was carried out by the author in the laboratory of the American Harmonica Company in Chicago, Illinois, in the years 1939–1941, and subsequently in the plant of John Luellen and Company in Hazel Crest, Illinois, in 1944.

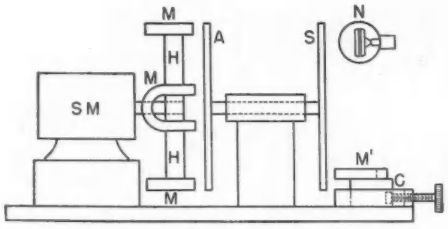

FIG. 1. Schematic diagram of the pitch comparator for tuning harmonica reeds.

mounted. A stroboscopic device was developed, which could be operated successfully by relatively unskilled personnel, the ear thus being replaced by the eye, and the burden of judging the pitch taken over by a scientific instrument.

This tuning indicator or pitch comparator consisted (Fig. 1) of a stroboscopic disk S mounted on the same shaft (journaled in ball bearings not shown) with an aluminum (eddy current or armature) disk A driven by an electromagnetic clutch. The latter comprised six alnico horseshoe magnets M, mounted on the rim of an aluminum hub H, which in turn was mounted on the shaft of a synchronous motor SM. The sixty-cycle frequency of the Chicago power network was relied on for constancy. While this is said to be constant to one part in a million on the average over a period of a day, instantaneous fluctuations occur which depress considerably this high order of constancy. An improvement would be to drive the synchronous motor by means of a generator of constant frequency such as an electrically driven tuning fork, crystal oscillator, etc., suitable amplification being interposed between generator and motor.

The pole faces of the magnets were placed parallel to, but out of contact with, the armature disk. On rotation the magnets then produced an electromagnetic drag on this disk, causing it to rotate at a constant speed, which depended, however, on its distance from the magnets. The fine adjustment of the speed was made by providing an electromagnetic brake, comprising a single alnico magnet M' mounted on a carriage C fitted with a micrometer screw, so that the pole faces of the magnet could be moved toward or away from the armature disk. With the latter arrangement the speed of the stroboscopic disk could be adjusted accurately to any desired pitch. The stroboscopic disk itself carried a number of stroboscopic circles[2] or patterns, so that a change-over from one note to the other could be quickly made. The speed of the apparent rotation of the pattern gave a measure of the pitch discrepancy, unison or coincidence being indicated by a stationary pattern.

The reed to be tuned was sounded by clamping tightly the reed plate (with the reeds uppermost) to the top of an airtight box, provided with a single slot slightly larger than the slot in the reed plate. A slide valve controlled the opening or closing of this slot. The interior of the box was connected to a vacuum system, which provided up to an inch of water differential of suction. The latter was produced by inverting a cylindrical sheet-iron tank U (Fig. 2) inside a similar upright tank L of slightly larger diameter, filled with water, and attaching a weight W, of magnitude determined by the desired vacuum, by means of a rope and pulley system so that an upward pull was produced on the inverted tank. A pipe P led from the interior of the inverted tank to the sounding box. (It is necessary to provide a constant pressure differential for sounding the reeds because the note emitted varies slightly with the pressure.) This simple system, which served its purpose admirably, was subse-

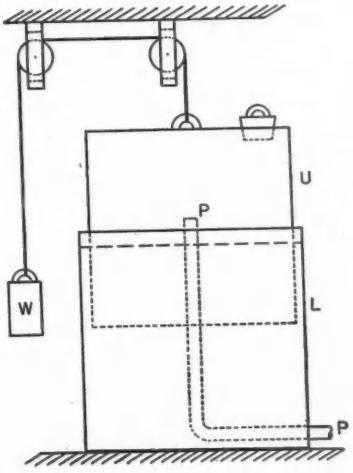

FIG. 2. Schematic diagram of the constant pressure device for sounding the reeds.

[2] A stroboscopic disk supplied by the Central Scientific Company was used.

quently replaced by a suitable blower system, which required less space. The tanks described were about 30 in. in diameter and 36 in. high. The "weight" was simply a pail filled with gravel to the proper level.

The pressure differential required to sound the reeds was measured by attaching an inclined tube manometer, with a range of one inch of water, to the sounding box. The required differential increases with the pitch, reaching a value of about 0.75 inch of water for the highest note of the C harmonica. The goal of the manufacturer is to make the pressure differential required to sound the reeds as low as possible, that is, to produce an "easy blowing" instrument.

Inside the sounding box and directly under the reed was placed a crystal microphone (simply one of a pair of crystal earphones), from which a microphone cable led to the input of an 8-watt audio amplifier. The output of the amplifier was connected to a neon glow tube N (see Fig. 1), mounted in front of the stroboscopic disk. The procedure was to adjust the speed of the stroboscopic disk to the pitch of a standard reed and then to substitute the unknown. The apparent speed of rotation gave a measure of the discrepancy in pitch between the unknown and the standard, while the direction of the apparent rotation indicated whether the unknown was higher or lower than the standard. Since the pitch of the reeds changes slightly when the reed plate is subsequently attached to the block, the standard is adjusted to compensate for this change in pitch.

This compensation is automatically achieved by tuning a selected reed plate, while mounted on a harmonica block, to the correct pitch (by comparison with tuning forks or other frequency standards) and then, for use as a standard, removing the plate from the block. The reeds are tuned in production with the reed plate off the block for convenience in handling and also because the blow reeds (facing inward) would be accessible on the block only with difficulty. Both "blow" and "draw" reeds can be sounded as "draw" reeds on the sounding box by the simple expedient of turning the "blow" reed plate over so that it will function as a "draw" plate (suction) during test.

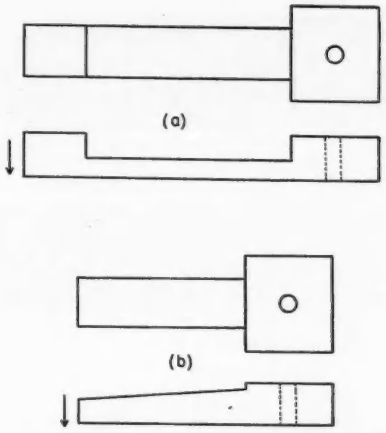

FIG. 3. Shapes of harmonica reeds: (a) low pitch reeds (b) high pitch reeds. The arrow indicates the direction of vibration.

2. MANUFACTURE OF THE REEDS

The ordinary 10-hole (nonchromatic) harmonica of the "Marine Band" type encompasses three octaves. The scale proper, however, begins with the fourth hole, the first three being arranged to sound on "blow" the major chord of the key to which the harmonica is tuned, and on "draw" the dominant one. For a harmonica tuned to the key of C, the actual arrangement is as follows:

Hole	1	2	3	4	5	6	7	8	9	10
Blow	C	E	G	C	E	G	C	E	G	C
Draw	D	G	B	D	F	A	B	D	F	A

The lowest C corresponds to middle C of the piano. It will be noted that the note B is missing in the third octave. The reeds are composed of so-called naval brass rolled to a spring temper. The reeds are uniform in width but of two types of cross section, namely, those shown in Figs. 3(a) and 3(b). In the key-of-C harmonica the reeds in holes 1, 2, and 3 "blow" and in 1 and 2 "draw" have the section shown in Fig. 3a, the pitch being varied by changing the total length, the thickness of the middle (flat) section, and the length of the offset portion; while the pitch of the other reeds is varied by changing the total length and the taper. The length of the reed is, of course, fixed by the (predetermined) length of the corresponding slot, thus leaving the taper

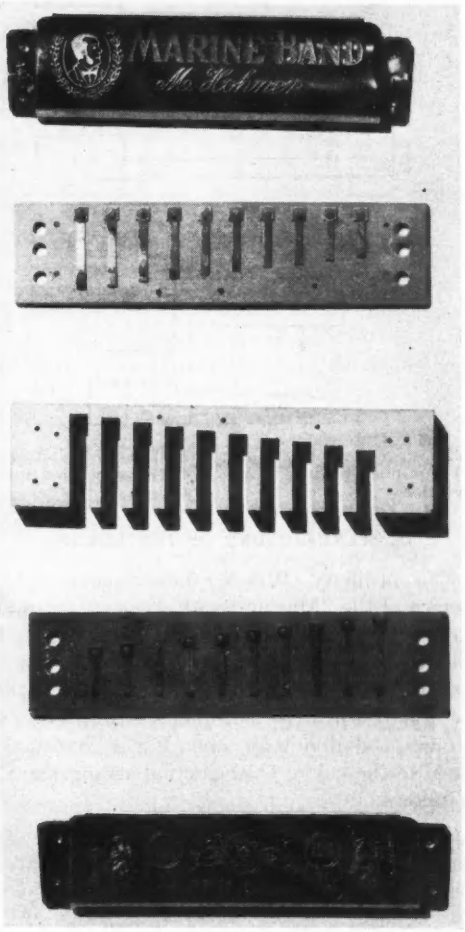

FIG. 4. Exploded view of harmonica.

in the case of reeds of type of Fig. 3(b) as the only variable in manufacture. The correct taper is determined by the desired pitch. The successive slots are staggered in length apparently largely on an empirical basis developed as a compromise over years of manufacture. The density and Young's modulus of brass rolled to a spring temper are fairly constant. However, close metallurgical control is desirable. The correct taper is best determined experimentally.[3]

[3] The vibration frequencies of reeds of constant cross section and of those tapered in width (but not in thickness) are discussed in Morse, *Vibration and Sound* (McGraw-Hill Book Company, Inc., New York, 1936), pp. 111–127, especially p. 126. Accordion reeds are tapered both in thickness and in width to obtain greater range. Harmonica reeds, however, cannot be treated as reeds mounted on a massive plate. In fact, as pointed out above, the pitch of the reeds changes slightly even when the reed plate is mounted on the harmonica block. Also, the pitch of the reeds varies slightly with the pressure at which they are blown. Hence theoretical calculations would give only approximate results.

The taper in the case of the reeds of the type of Fig. 3(b), as well as the flat portion of the reeds of the type of Fig. 3(a), is produced by milling lengthwise a strip of flat sheet brass, from which the reeds are stamped out subsequently on a punch press. The reeds are cut so that the long axis is parallel to the direction of rolling and are mounted on the reed plate by riveting. In these two operations, milling the strips and mounting the reeds, lies practically the whole secret of successful and economical manufacture. The taper must be accurately controlled by controlling the thickness to 0.0001 inch, so that the amount of tuning subsequently required be reduced to a minimum; and the reeds must be accurately aligned in their respective slots with only a clearance of 0.0015 inch between reed and slot. The latter clearance is highly important because it determines largely the ease of blowing, which is the only figure of merit difficult of achievement. Good tone quality and fairly long reed life are relatively easy to obtain.

3. ASSEMBLY OF THE HARMONICA

The reed plates (one carrying the "blow" reeds, the other the "draw" reeds) are mounted one on each side of a central wooden block, which is provided with ten evenly spaced slots of depths corresponding to the various reed lengths (Fig 4). The "blow" reeds necessarily face inward, the "draw" reeds, outward. The only characteristics required of the wood are that it be close-grained so as to be unaffected by moisture and that it take without splitting the small nails used to hold the reed plates to the block. The wood used in German instruments usually is apple or pear wood, but, because of the difficulty of obtaining the former, cherry was tried instead and found to give satisfactory results. The block itself requires rather accurate manufacture—at least an accuracy higher than that ordinarily used in wood working, to assure uniform thickness for

airtightness and accurate slot spacing. The addition of two nickel-plated, suitable-shaped "covers" completes the instrument.

In conclusion it might be noted that the problem confronting an American manufacturer is not how to produce a better harmonica but how to produce a harmonica at a cost which will meet European competition. This poses interesting problems for the physicist, engineer, and metallurgist.

Count Rumford's Concept of Heat*

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Rumford developed his concept of heat by analogy with the acoustic waves emitted by a bell. He felt that temperature was a measure of the frequency of vibration of the emitting source; the higher the temperature, the higher the frequency. This led him also to the belief in the existence of frigorific rays as separate entities from calorific rays.

CLIMAXING his famous paper,[1] "An Inquiry Concerning the Source of the Heat which is Excited by Friction," Count Rumford wrote: "it appears to me to be extremely difficult, if not quite impossible, to form any distinct idea of anything capable of being excited and communicated in the manner the Heat was excited and communicated in these experiments, except it be MOTION."

It is of interest to inquire what Rumford meant by "motion" in this often-quoted passage, and to find that intuitively he accepted an interesting mixture of a remarkably modern theory of heat in solids with the concept that temperature was associated with the frequency of an oscillating source.

When Benjamin Thompson was seventeen years of age, and furthering his education as best he could by reading everything available, he came across Boerhaave's "Treatise on Fire."[2] Boerhaave, whose books on natural philosophy were standard texts in the eighteenth century, gave great weight to the theory that heat, like sound, was a product of the vibration of the heated body. Thompson accepted this point of view very early in his life. Almost never did he give credit to previous scientific work, but one of the very few references he did cite[3] in his many volumes of writings was to Boerhaave's "Chemistry" in which this author likens a heated body to a struck bell, an analogy which the Count uses constantly.[4] "Suppose now that heat be nothing more than the motions of the constituent particles of bodies among themselves (an hypothesis of ancient date, and which always appeared to me to be very probable), if for the bell we substitute a hot body, the cooling of it will be attended by a series of actions and reactions exactly similar to those just described.

"The rapid undulations occasioned in the surrounding ethereal fluid, by the swift vibrations of the hot body, will act as calorific rays on the neighbouring colder solid bodies, and the slower undulations, occasioned by the vibrations of those colder bodies, will act as frigorific rays on the hot body; and these reciprocal actions will continue, but with decreasing intensity, till the hot body and those colder bodies which surround it shall, in consequence of these actions, have acquired the same temperature, or until their vibrations have become isochronous."

The experiments which led Thompson to the vibratory theory of heat were those which he performed in England in 1778 on the heating of guns during firing.[3] He found that guns fired without bullets always got hotter than those

* Supported in part by a grant from the Penrose Fund of the American Philosophical Society.
[1] Count Rumford, Trans. Roy. Soc. (London) **88**, 80 (1798).
[2] Count Rumford, *Works of Count Rumford* (American Academy of Arts and Sciences, Boston, 1873), vol. II, p. 188.
[3] B. Thompson, Trans. Roy. Soc. (London) **71**, 229 (1781).
[4] Count Rumford, Trans. Roy. Soc. (London) **94**, 77 (1804).

which fired shot. In attempting to explain these observations he reasoned that the expanding gases resulting from the explosions passed through the gun with a higher velocity when they were free than when they were impelling a ball before them. He went on to conclude that the higher velocity explosion produced a higher frequency oscillation of the material of the gun, and hence a higher temperature.

The ordinary phenomena associated with heat were easily explained by this theory. Thermal expansion and contraction resulted form an increased or decreased amplitude of oscillation of the constituent particles which made up the body. The oscillatory theory postulated a maximum amplitude of oscillation of the particles that made up a body which, if exceeded, caused a change in phase. The latent heat given up when a liquid froze was taken as proof that the vibratory motion resided also in the liquid and became available for measurement when the liquid solidified. Incidentally, Rumford rejected the concept of an absolute zero of temperature with the following words:[4] "*Hot* and *cold*, like *fast* and *slow*, are mere relative terms; and, as there is no relation or proportion between motion and a state of rest, so there can be no relation between any degree of heat and absolute cold, or a total privation of heat; hence it is evident that all attempts to determine the place of *absolute cold*, on the scale of a thermometer, must be nugatory. It seems probable that *motion* is an essential quality of matter and that rest is nowhere to be found in the universe."

One of the attractive features of the vibratory theory was that the same mechanism applied in conduction as in radiation of heat. He wrote:[5] "A theory which should have the advantage of explaining the communication of heat by a *single* method, at once simple and easy to understand, would be preferable, it seems to me, to one which, in order to explain various phenomena, would be obliged to admit *two different* modes of the communication of heat." Rumford recognized no difference between acoustic and optic waves and was led to make the shrewd observation that[4] "there are so many striking analogies between the rays of light and those invisible rays which all bodies at all temperatures appear to emit, that we can hardly doubt of their motions being regulated by the same laws. Perhaps there may be no other difference between them than exists between those vibrations in the air which are audible and those which make no sensible impression on our organs of hearing."

The Count's concept of heat is well illustrated in his introduction to his cannon boring experiment:[2] "I argued that if the existence of caloric was a fact, it must be absolutely impossible for a body or for several individual bodies, which together made one whole, to communicate this substance continuously to various other bodies by which they were surrounded, without this substance gradually being entirely exhausted. A sponge filled with water and hung by a thread in the middle of a room filled with dry air, communicates its moisture to the air, it is true, but soon the water evaporates and the sponge can no longer give out moisture. On the contrary, a bell sounds without interruption when it is struck, and gives out its sound as often as we please without the slightest perceptible loss. Moisture is a substance; sound is not. It is well known that two hard bodies, if rubbed together, produce much heat. Can they continue to produce it without finally becoming exhausted? Let the result of experiment decide this question."

Rumford's conviction that each temperature corresponded to a particular frequency of oscillation of a body led him to champion actively the concept of "calorific" and "frigorific" rays first postulated by Plutarch in his *De Primo Frigido*. Ever since Jean Baptista Porta in 1590 conducted experiments showing the focusing of radiation from cold bodies by concave mirrors, those who repeated these experiments found it difficult not to admit the existence of frigorific rays. During the 1780's Rumford's intimate friend and fellow philosopher, Professor Pictet of Geneva, carried out a long series of experiments on the focusing of cold which so intrigued the Count that he also conducted many tests to show that frigorific radiation existed.

In the autumn of 1800, while visiting Scotland, Rumford, with three professors from the University of Edinburgh (Hope, Playfair, and Stewart) conducted a typical experiment of this kind:[2] "Two metallic mirrors fifteen inches in

[5] Count Rumford, *Moniteur Universal*, 9 Messidor, An 12 (June 26, 1804).

diameter, with a focal distance of fifteen inches, were placed opposite each other, sixteen feet apart. When a cold body (for example, a glass bulb filled with water and pounded ice) as was the case on this occasion, was placed in the focus of one of the mirrors, and a very sensitive air-thermometer was placed in the focus of the other mirror, the latter thermometer began immediately to fall. If, instead of being placed directly in the focus, the thermometer was removed a short distance from it to one side, the cooling power which in the former case the cold body had exerted upon it was no longer perceptible." As Rumford wrote[6] to Pictet in Geneva on October 18, 1800, "We repeated your interesting experiment on the reflection of cold, two days ago at Dr. Hope's house, and with complete success ···. The slow vibrations of ice in the bottle cause the thermometer to sing a lower note."

Count Rumford equipped his Royal Institution with four large mirrors for demonstrating the focusing of "calorific" and "frigorific" rays. The use of these is illustrated in the figure taken from John Tyndall's "Heat Considered as a Mode of Motion."[7] Tyndall was Professor of Natural Philosophy in the Royal Institution and his demonstration lectures in physics have survived for almost a hundred years as a model for many similar survey courses. In the figure he is shown exploding a hydrogen-oxygen balloon by means of Rumford's mirrors, but in the next experiment he says, "I again lower the mirror, and, ··· suspend a [flask] containing a freezing mixture. I raise the mirror and, as in the former case, bring the [thermo] pile into the focus of the lower one. Turned directly toward the upper flask there is no action; turned downwards, the needle moves: observe the direction of the motion—the red end curves toward me [indicating cold]. Does it not appear as if this body in the upper focus were now emitting rays of cold which are converged by the lower mirror exactly as the rays of heat in our former experiment. The facts are exactly complementary, and it would seem that we have precisely the same right to infer from the experiments, the existence and convergence of these cold rays, as we have to infer the existence and convergence of the heat rays."

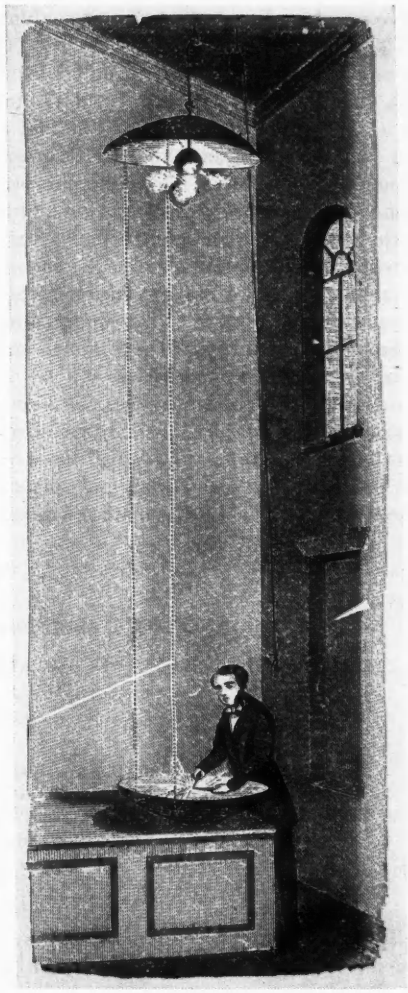

FIG. 1. Royal Institution mirrors for focusing heat and cold.

Count Rumford's belief that cold was a separate entity from heat was a very real one. In discussing the radiation of cold he wrote:[4] "According to this hypothesis, *cold* can with no more propriety be considered as the absence *of heat* than a low or grave sound can be considered as the absence of a higher or more acute note; and the admission of rays which generate cold

[6] Manuscript Collection of the American Academy of Arts and Sciences, Boston, Massachusetts.

[7] John Tyndall, *Heat Considered as a Mode of Motion* (D. Appleton-Century Company, Inc., New York, 1871).

involve no absurdity and create no confusion of ideas."

He saw in the workings of nature confirmation of his ideas. Were not people black in hot climates and white in cold ones? The black skin of the Negro allowed him to radiate efficiently and thereby keep cool, while white skin was an efficient reflector of frigorific radiation and hence defended the white man from the cold. Rumford was so sure of his conclusions that he carried his convictions to the logical conclusion of always wearing white clothing in cold weather, much to the derisive amusement of the Parisian society in which he moved in later years.

Rumford had no concept of heat as a random motion. He felt that heat was primarily set up by the harmonic vibrations of the "fibers of the metal,"[7] was transmitted through solids and radiated from them in the same manner as acoustic waves. He did not feel that these same waves could be set up in fluids. In fact, he carried out a long series of experiments[8] showing that gases and liquids (including mercury!) were perfect nonconductors and that their only mode of communicating heat was by convection.[9] He felt that what heat was transmitted through fluids at rest was due only to the conduction of thermal vibrations in the all-pervading ether, and he was strengthened in this belief by his showing that heat passed almost as easily through a Torricellian vacuum as through air.[10] By 1800 he was completely convinced that heat was a vibratory motion, analogous in every way to acoustical oscillations.

[8] Count Rumford, Essay VII, Part II, Cadel and Davies, London, 1798.
[9] S. C. Brown, Am. J. Phys. **15**, 273 (1947).
[10] Count Rumford, Trans. Roy. Soc. (London) **76**, 273 (1786).

A Direct Treatment of the Foucault Pendulum

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The equations of motion of the Foucault pendulum are set up in polar coordinates. The oscillation is shown to be simple harmonic for a particular angular velocity, $-\Omega \sin\phi$, where Ω is the angular velocity of rotation of the earth and ϕ is the latitude. In general, the motion involves a constant areal velocity c and a nonlinear oscillation given by $\ddot{r}+\omega^2 r-c^2/r^3=0$. This equation is integrated through the energy equation and shown to give the same precession as in the harmonic case.

FOUCAULT'S great pendulum of 1851, swinging under the dome of the Panthéon in Paris, gave a dramatic proof of the rotation of the earth. The slow precession of the plane of oscillation of a freely suspended pendulum is still followed with interest, when the experiment is repeated in observatories and science museums throughout the world, and many elementary physics textbooks discuss the phenomenon as a laboratory demonstration of the earth's rotation. As has been pointed out recently,[1] it is not correct to say simply that the pendulum continues to swing in a fixed plane or that the horizontal path of the bob maintains a fixed direction in space. It is quite possible to give a reasoned geometrical explanation[2] of the observed precession of the plane of oscillation, but this is cumbersome and rarely done.

The more advanced books offer a variety of treatments of the Foucault pendulum. With the traditional method of starting the oscillations, by burning a string which holds the bob out from the equilibrium position, the motion should follow a pointed-star pattern as described in a detailed derivation[3] of the special cases by Kimball in this journal. The author points out that, in practice, his different patterns are indistinguishable and amount to a simple harmonic motion in a slowly rotating plane. A complete

[1] Wylie, Pop. Astron. **57**, 170 (1949).
[2] Grimsehl, *A Textbook of Physics* (Blackie & Son Limited, London and Glasgow, 1932), Vol. 1, p. 165.
[3] W. S. Kimball, Am. J. Phys. **13**, 271 (1945).

development[4] from the differential equations of the spherical pendulum shows the dependence on the initial conditions. This derivation, a concise vector treatment,[5] and a development in complex quantities[6] have in common the introduction of a rotation of the reference axes to compensate for the rotation of the earth. There follows an alternate direct treatment of this historic and important example of motion on a rotating earth, quite within the reach of students of intermediate mechanics.

THE CORIOLIS FORCE

By the theorem of Coriolis (1829), motion relative to moving axes can be treated by the ordinary equations of motion by the addition, to the actual forces, of fictitious forces capable of producing accelerations equal and opposite to the acceleration of moving space and the compound centripetal acceleration. This is, of course, an application of the principle, introduced by d'Alembert in his *Traité de Dynamique* (1743), that the system of external forces is, as a whole, in dynamic equilibrium with the inertial reactions of the accelerated masses. For the dynamics of a particle, d'Alembert's principle is implicit in Newton's second law (1687) and the theorem of Coriolis involves only the kinematics of relative motion. A modern derivation can be found in any standard textbook.

For an observer on the uniformly rotating earth, the first fictitious force reduces to the ordinary centrifugal force of elementary mechanics. It is constant at a given station and is included in the resultant gravitational force, the weight of the body. The second, the Coriolis or deflecting force, depends on the latitude and the velocity of the body; it is familiar to many physics students through its dominant role in the calculation of the geostrophic wind velocity in meteorology. For motion in a horizontal plane, the horizontal component of the Coriolis force is at right angles to the velocity v of the body and has a magnitude $2mv\Omega \sin\phi$ where, for a latitude ϕ, $\Omega \sin\phi$ is the vertical component of the angular velocity of the earth.

[4] Webster, *The Dynamics of Particles and of Rigid, Elastic, and Fluid Bodies* (B. G. Teubner, Leipzig, Germany, 1904), p. 323.

[5] Page, *Introduction to Theoretical Physics* (D. Van Nostrand and Company, Inc., New York, 1935), second edition, p. 107.

[6] Joos, *Theoretical Physics* (Blackie & Son Limited, London and Glasgow, 1951), second edition, p. 814.

THE EQUATIONS OF MOTION

With a long suspension and a small maximum displacement from the equilibrium position, the motion of a pendulum bob is effectively in the horizontal plane. The simple pendulum of mass m, length l, and small amplitude of oscillation is assumed to execute simple harmonic motion under the influence of a restoring force, mgr/l, proportional to the displacement r. As this approximation is always introduced sooner or later, we shall distinguish the Foucault pendulum proper from the spherical pendulum by postulating its validity in what follows.

Using polar coordinates for the position of the pendulum bob in the horizontal plane, we have at any point radial and transverse velocities, $\dot{r}$ and $r\dot{\theta}$, and the corresponding accelerations, $\ddot{r}-r\dot{\theta}^2$ and $r\ddot{\theta}+2\dot{r}\dot{\theta}$. With the inclusion of the Coriolis forces, the equations of motion are

$$m(\ddot{r}-r\dot{\theta}^2) = -mgr/l+2mr\dot{\theta}\Omega \sin\phi \qquad (1)$$

and

$$m(r\ddot{\theta}+2\dot{r}\dot{\theta}) = -2m\dot{r}\Omega \sin\phi. \qquad (2)$$

THE HARMONIC OSCILLATOR

The equations of motion reduce to

$$\ddot{r}+r(g/l-\dot{\theta}^2-2\dot{\theta}\Omega \sin\phi) = 0 \qquad (3)$$

and

$$r\ddot{\theta}+2\dot{r}(\dot{\theta}+\Omega \sin\phi) = 0. \qquad (4)$$

Equation (4) is satisfied by a constant angular velocity, $-\Omega \sin\phi$, of the pendulum bob since $\dot{\theta}+\Omega \sin\phi = 0$ gives $\ddot{\theta} = 0$. Substituting this value for $\dot{\theta}$ in Eq. (3), we get

$$\ddot{r}+\omega^2 r = 0, \qquad (5)$$

where $\omega^2 = g/l+\Omega^2 \sin^2\phi$. This represents a simple harmonic motion along a radius vector with a period T given by

$$T = 2\pi(g/l+\Omega^2 \sin^2\phi)^{-\frac{1}{2}}. \qquad (6)$$

For motion from a maximum displacement A at an angle θ_0, the complete solution is given by

$$r = A \cos(g/l+\Omega^2 \sin^2\phi)^{\frac{1}{2}}t \qquad (7)$$

and

$$\theta = \theta_0-\Omega t \sin\phi. \qquad (8)$$

THE NONLINEAR OSCILLATOR

In general, the solution of Eq. (4) is of the form

$$r^2(\dot{\theta}+\Omega\sin\phi)=c, \tag{9}$$

where c is a constant areal velocity. (This is not the rate at which the radius vector sweeps out area, which is simply $\frac{1}{2}r^2\dot{\theta}$.) Substituting from Eq. (9) into Eq. (3), we get the equation of a nonlinear oscillator,

$$\ddot{r}+\omega^2 r-c^2/r^3=0. \tag{10}$$

This is not one of the common nonlinear differential equations, but it is of interest to apply to it some of the more recently developed techniques for the solution of such equations.[7] Fortunately, in the present case, we can turn to the energy equation as a first integral of the equation of motion and can integrate it in terms of a new variable.

Since the Coriolis force is at right angles to the velocity and does no work, the energy equation is simply

$$\tfrac{1}{2}m(\dot{r}^2+r^2\dot{\theta}^2)+\tfrac{1}{2}mgr^2/l=\text{constant}. \tag{11}$$

At the maximum displacement A, the radial velocity $\dot{r}$ is zero and we can substitute from Eq. (9) and evaluate the constant. We find

$$\dot{r}^2+\omega^2r^2+c^2/r^2=\omega^2B^2, \tag{12}$$

where $B^2=A^2+c^2/\omega^2A^2$. This equation can be written in terms of a new variable z, where $z=r^2$ and $\dot{z}=2r\dot{r}$, and becomes

$$\dot{z}^2=-4c^2+4\omega^2B^2z-4\omega^2z^2. \tag{13}$$

This can be integrated and gives the solution of the nonlinear equation as

$$r^2=\tfrac{1}{2}B^2+\tfrac{1}{2}C^2\cos2\omega t, \tag{14}$$

where $C^2=A^2-c^2/\omega^2A^2$.

[7] See, for example, Schelkunoff, Quart. Appl. Math. **3**, 348 (1945).

THE PRECESSION OF THE PENDULUM

The angular velocity $-\Omega\sin\phi$ required for harmonic oscillation along the radius vector is small. It is sometimes argued[8] that the bob may well acquire this angular velocity in starting from the rest position. That this is not the explanation of the regularly observed precession is seen by reflecting that a masking angular velocity c/A^2 is equally probable. The student must consider the effect of an arbitrary areal velocity c on the pendulum motion. Substituting from Eq. (14) in Eq. (9), we have

$$\dot{\theta}=2c/(B^2+C^2\cos2\omega t)-\Omega\sin\phi. \tag{15}$$

This is integrable and, writing $\Delta\theta$ for the angle turned through in one complete oscillation of the pendulum, we have

$$\Delta\theta=-\Omega T\sin\phi. \tag{16}$$

Thus, the precession of the pendulum is shown to be $-\Omega\sin\phi$ in the anharmonic as in the harmonic case.

The general solution of the equations of motion enables us to calculate the position of the pendulum bob for any given initial conditions. In particular, from Eq. (14), we obtain the minimum displacement from the equilibrium position as $c/\omega A$. This gives at once the simple condition, $c=0$, that the bob pass through the equilibrium position. From Eq. (9), we see that under no condition can the Kepler law of equal areas be obeyed.

Erratum: Quantitative Evaluation of Rocket Propellants

S. S. PENNER
Guggenheim Jet Propulsion Center, California Institute of Technology, Pasadena, California
[Am. J. Phys. **20**, 26 (1952)]

EQUATION (12) should read

$$c^*=(1/\lambda)(RT_c/\delta\bar{M}^*)^{\frac{1}{2}}. \tag{12}$$

Reproductions of Prints, Drawings, and Paintings of Interest in the History of Physics

46. Portrait of Christiaan Huygens by C. Netscher

E. C. Watson
California Institute of Technology, Pasadena, California
(Received June 2, 1952)

This note points out that excellent reproductions in color of Netscher's fine portrait of Christiaan Huygens may be purchased at small cost from the Municipal Museum, the Hague.

FIVE portraits of Christiaan Huygens (1629–1695) are reproduced in the monumental *Oeuvres complètes de Christiaan Huygens* published by the Société Hollandaise des Sciences. These are the following:

1. The portrait of Constantijn Huygens and his five children, painted in 1640 by Adriaen Hanneman, which now hangs in the art gallery at the Hague.
2. The engraved portrait by Edelinck.
3. The medallion executed by J. Clérion in 1679.
4. The portrait painted in 1687 by P. Bourguignon.
5. The portrait painted in 1671 by Caspar Netscher (1639–1689).

The portrait by Netscher is in some respects the most interesting. It was completed two years before the publication of the *Horologium oscillatorium* and shows Huygens in his prime. He was at the time living in Paris, having been invited by Colbert on behalf of Louis XIV to be a member of the newly formed Academy of Sciences, where he occupied an influential, if not a dominant, position.

The reason for reproducing this portrait in this series is to point out that the original hangs in the Gemeentemuseum, s-Gravenhage, and that excellent reproductions *in color* and of a size (approximately 9 in. by 7 in.) suitable for framing can be obtained at very small cost from the following address:

Dienst voor Kunsten en Wetenschappen der Gemeente
Directie van het Gemeentemuseum
Stadhouderslaan 41
Den Haag.

The soft and beautiful colors of the original portrait add greatly to its effectiveness.

Fig. 1. Christiaan Huygens at the age of forty-two.

Bodies Rocking on Planes and Cylinders, Spheroids and Spindles Rocking on Planes, Balls and Other Symmetrical Bodies Rocking in Elliptical and Other Holes

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(Received November 19, 1951)

In continuance of earlier work (references below) the rocking of bodies of other shapes resting on different supports has been studied. The examples afford a useful study of the dynamical equations appropriate to such vibrations and provide simple and inexpensive experiments for students in the laboratory. Rocking bodies are often observed in every day occurrence, and interest in such motion is well worth development.

IF a body vibrates under the action of gravity and its center of gravity is below the point of support, the body may be said to *swing* to-and-fro, e.g., an ordinary pendulum. If the body rests on a horizontal plane (or partially in a hole in the plane) so that its center of gravity is above the support and it vibrates under the action of gravity about an equilibrium position it may be said to *rock*, e.g., a rocking-chair or a rounded pebble on a flat surface.

In two previous papers in this journal[1] I have given descriptions of experiments on rocking bodies with the determination of their periods of rocking; experiments that are suitable for students in mechanics and physics laboratories. These experiments were based upon two problems in Lamb's *Dynamics*.[2]

The first problem is: "A circular cylinder of radius a whose mass-center is at a distance b from the axis rolls on a horizontal plane."

Lamb shows that, if the amplitude is small, the period of vibration of the to-and-fro rocking moment is given by

$$T=2\pi[(k^2+(a-b)^2)/(bg)]^{\frac{1}{2}}, \qquad (1)$$

where k is the radius of gyration of the whole body about an axis through the mass-center parallel to the axis of the cylinder and g is the acceleration of gravity. Lamb adds, "The result evidently applies to any case of a solid of revolution rolling parallel to a vertical plane of symmetry at right angles to the axis."

If L is the length of the equivalent simple pendulum we may write

$$L=[k^2+(a-b)^2]/b. \qquad (2)$$

The second problem is: "A cylinder, of any form of section, rocks on a horizontal plane making small oscillations about a position of equilibrium. Find the period of rocking."

Lamb shows that in this case

$$L=(k^2+h^2)/(R-h), \qquad (3)$$

where k is the radius of gyration of the cylinder about an axis through its mass-center perpendicular to the plane of rocking, h is the distance of the mass-center from the point of contact corresponding to the stable position, and R is the radius of curvature of the curve at the same point of contact.

In the October, 1950 paper, I illustrated the first problem with incomplete circular bands, solid segments of circles, hollow and solid segments of spheres and the second problem with a solid ellipse. In the December, 1951 paper an elliptical band was studied.

In practical work it is convenient to remember that since $T=2\pi(L/g)^{\frac{1}{2}}$, the value of g in Toronto is such that this reduces to $T=0.2006(L)^{\frac{1}{2}}$, when cgs units are used, so that it is sufficient to use the relation $T=\frac{1}{5}(L)^{\frac{1}{2}}$ when testing the agreement between calculation and observation. In most cases the motion is simple harmonic only for very small amplitudes, but timing has to be made with finite amplitudes. Also, unless the surfaces are true to shape and hard there is no advantage in taking the amplitude very small, for this is likely to make the period of vibration dependent on small imperfections of the body and its support near the normal point of contact. In some cases it is interesting to extend the timing observations over a wide range of amplitude in order to find how the period depends on

[1] Am. J. Phys. **18**, 405 (1950); **19**, 54 (1951).
[2] H. Lamb, *Dynamics* (Cambridge University Press, London, England, 1923), 2nd edition, p. 188.

the amplitude and thus to obtain a fuller solution of the problem. The observations are approximate only, they are recorded as they are taken, and no attempt has been made to apply least squares to reduce errors. A Venner stop-watch whose second hand makes a complete revolution in 10 seconds was used for the timing.

For the benefit of those who have not read my two previous papers, I shall begin the present paper with two simple cases of rocking on circular arcs and follow with accounts of the rocking of spheroids and a spindle on a horizontal table; the rocking of balls in an elliptical hole, in a hole formed by two circular arcs, and in a long double V-shaped slot of straight sides; the rocking of a double cone in the same holes, of the spindle in elliptical holes, and of circles and ellipses rocking on the top of or within circles and ellipses. Examples may be continued indefinitely, but one must not tire the editor nor his readers. I hope to persuade the latter to try such experiments for themselves.

1. ROCKING OF A RING WITH AN ECCENTRIC LOAD

A simple piece of apparatus for this experiment is made by cutting a ring and a (smaller) disk from the same plate of metal. The dimensions may be chosen to make the masses equal as this makes the calculations easier (see below). The ring is placed vertically and the disk rested within it; motion of the disk relative to the ring is prevented by little wedges of plasticine. In Fig. 1(a), O is the center of the ring, D the center of the disk; G is the center of mass of the whole. In my set-up $OG=GD$.

The period of rocking is calculated from Lamb's expression [Eq. (1)]. The dimensions of the ring are i.d. 8.00 cm, o.d. 10.00 cm; the diameter of the disk is 6.00 cm. Therefore, $OC=a=5.00$ cm, $CD=4.00$ cm, $OG=b=0.50$ cm, and $k^2=12.75$ cm^2. Thus

$$L=\frac{12.75+4.50^2}{0.50}=66 \text{ cm}$$

and

$$T(\text{calc.})=\tfrac{1}{8}L^{\frac{1}{2}}=1.62 \text{ sec.}$$

The period of rocking was observed in the usual manner. The recorded intervals for 10 rocks were 16.4, 16.3, 16.3, 16.3, 16.2, 16.3, 16.2, 16.3 sec with a mean 16.3 sec. Thus $T(\text{obs.})=1.63$ sec. The smoothness of surface and the accuracy of the lathe are responsible for such good agreement.

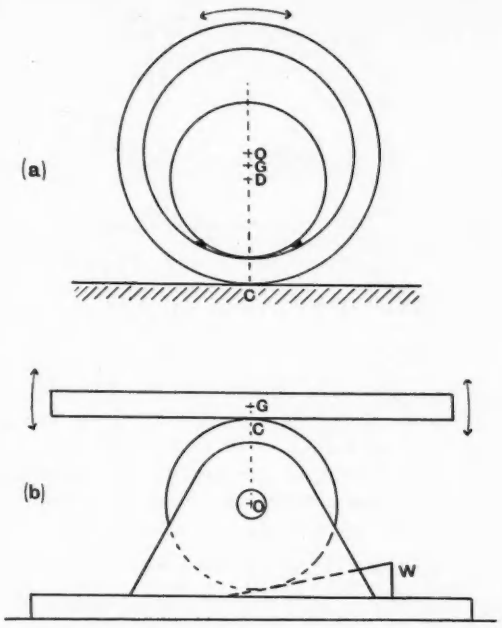


FIG. 1. (a) Ring with disk fixed excentrically rocking on a horizontal plane. Curved arrows indicate direction of rocking. (b) Uniform rod rocking on a fixed cylinder. Curved arrows indicate direction of rocking.

2. ROCKING OF A ROD RESTING UPON A FIXED CYLINDER

The calculation of the period of rocking is set as an exercise in Lamb's *Dynamics*. The experiment, with theory and practice, is described in Searle's *Experimental Harmonic Motion*[3] and also in an early paper by the author, so that the calculation of the period is not given here.

The apparatus is simple and can be built up without much trouble from the pulley-wheels and rods found in any workshop. We have used this experiment for over thirty years in laboratory classes and it has never lost its interest. The wheel is mounted axially and so that it may be turned around as required. It is fixed by a wedge [W see Fig. 1(b)]. With a well-made steel

[3] H. Lamb, *Dynamics* (Cambridge University Press, London, 1923), second edition, p. 212, Ex. 13. G. F. C. Searle, *Experimental Harmonic Motion* (Cambridge University Press, London, 1915), pp. 72–75. J. Satterly, Trans. Roy. Soc. Can. **25**, 85–92 (1921), Section III.

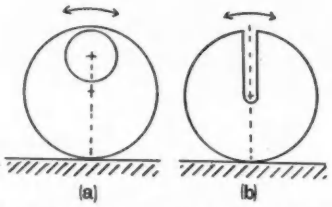


FIG. 2. (a) Disk with eccentric hole rocking on a horizontal plane. (b) Slotted weight rocking on a horizontal plane. Curved arrows indicate direction of rocking.

pulley-wheel of diameter about 20 cm and steel rods of rectangular cross section about 1 to 2 cm thick and lengths from 50 cm up to 100 cm good values of g may be obtained by the students.

Theory shows that if the cylinder has radius a, and the rod half-depth $d(d<a)$, and radius of gyration k about an axis through its center of gravity and perpendicular to the plane of rocking, the period of rocking is given by

$$T=2\pi\{(k^2+d^2)/(a-d)g\}^{\frac{1}{2}},$$

or the length of the equivalent simple pendulum by

$$L=(k^2+d^2)/(a-d). \qquad (4)$$

In the experiment for this paper, which I got ready in less than 5 minutes, the rod was a brass rod of length 30.5 cm and of 1.24 cm square section; the wheel was a brass disk of diameter 5.96 cm. Hence $k^2=77.5$ cm^2 and $b=0.62$ cm and L(calc.)$=33.1$ cm, whence T(calc.)$=1.15$ sec. The intervals for 10 vibrations, using different portions of the cylinder's edge and reversing the rod, were 11.3, 11.3, 11.4, 11.6, 11.5, 11.3, 11.7, 11.6, 11.6, 11.5, 11.8, 11.9, 11.6, 11.5, 11.6 with a mean of 11.55 sec. Hence T(obs.)$=1.155$ sec. The timing readings are given just as taken; no cooking was done with these or with any other of the observations reported in this paper.

An interesting case of a rod resting on a curve and not rocking is given by Sutton.[4] The curve is an inverted catenary and the d of the rod is equal to the c in the catenary equation $y=c\cosh(x/c)$. The rod is in neutral equilibrium.

3. ADDITIONAL VARIANTS OF SIMPLE ROCKING EXPERIMENTS

In addition to those given in my first paper, are the devices shown in Fig. 2. Figure 2(a) is a disk of metal with a circular portion cut out of it eccentrically, and Fig. 2(b) is an ordinary slotted "weight" as used with its fellows for loading scale pans in many physics experiments. In both cases theory and practice agree well. We shall meet the slotted weight again in a later section of this paper. A biconvex lens rocking on a smooth surface furnishes another interesting example; an egg, even if hard-boiled, is disappointing.

Note: The usual formula[5] for the period of swing of a compound pendulum containing the correction for the curvature of the knife-edge is

$$T=2\pi\{(k^2+h^2)/(h+r)g\}^{\frac{1}{2}},$$

where k^2 is as defined above, h is the distance from the point of support of the pendulum to the center of gravity of the pendulum, and r is the radius of curvature (supposed small) of the knife-edge. This is seen to be identical with Lamb's expression [Eq. (1)] above if we realize that $h=(b-a)$ and $r=a$ so that $h+r=b$.

4. ROCKING OF AN OBLATE SPHEROID

An oblate spheroid is the figure formed by the revolution of an ellipse about its shorter axis. Figure 3 shows different views of an oblate spheroid of semi-axes a, b, and c (axes a and b are horizontal, axis c is vertical, and $a=b$ and a and

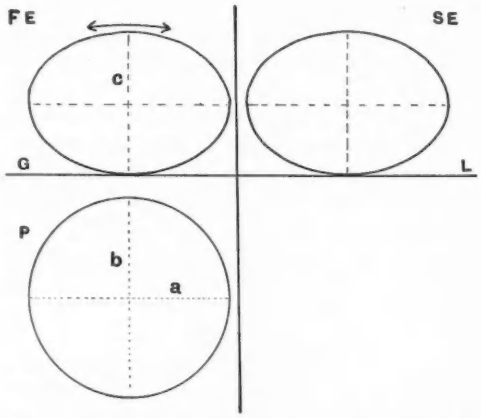


FIG. 3. Oblate spheroid rocking on a horizontal plane. GL is the ground line, and P, FE, SE are the plan, front elevation, and side elevation, respectively. Curved arrows indicate direction of rocking.

[4] Sutton, *Lecture Demonstrations in Physics* (McGraw-Hill Book Company Inc., New York, 1938), p. 27.

[5] Poynting and Thomson, *Properties of Matter* (Griffin and Company, London, 1947), fourteenth edition, p. 20.

b are both greater than c). Let the b axis be the axis of rocking. The calculus textbooks show that k^2 of this spheroid about the b axis is $\frac{1}{5}(a^2+c^2)$; also that R, the radius of curvature of the profile of the rocking curve at the equilibrium point of contact, is a^2/c. Hence by Lamb's expression [Eq. (3)]

$$L=\left[\tfrac{1}{5}(a^2+c^2)+c^2\right]\Big/\left[\frac{a^2}{c}-c\right]$$

$$=\frac{c}{5}(a^2+6c^2)/(a^2-c^2). \quad (5)$$

The oblate spheroid used had been made some years ago by gluing together slabs of wood to make a solid block, and this block was turned in the lathe to fit a tin-plate template. It was finally varnished to give a smooth surface. Dimensions: $2a=2b=16.6$ cm; $2c=14.3$ cm. Calculation gives $L=30.2$ cm and therefore T(calc.) $=1.10$ sec. The time of rocks was taken with a stop-watch graduated to tenths of a second; half way through, the spheroid was turned over. The intervals were 10.6, 10.7, 10.5, 12.0, 10.9, 11.0, 10.2, 10.7, 11.2, 11.7, 10.5, 10.6, 10.9. No reading has been rejected. Mean T(obs.) $=1.09$ sec. The agreement is fair.

5. ROCKING OF A PROLATE SPHEROID

A prolate spheroid is the figure produced by the revolution of an ellipse about its longer axis. Figure 4 shows different views of a prolate spheroid of semi-axes a, b, and c (axes a and b are horizontal and $a>b$; axis c is vertical and $b=c$). The b axis is the axis of rocking.

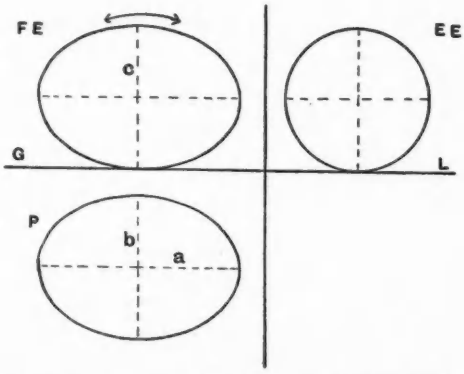


FIG. 4. Prolate spheroid rocking on a horizontal plane. *GL* is the ground line and *P*, *FE*, *EE* are the plan, front elevation, and end elevation, respectively. Curved arrows indicate direction of rocking.

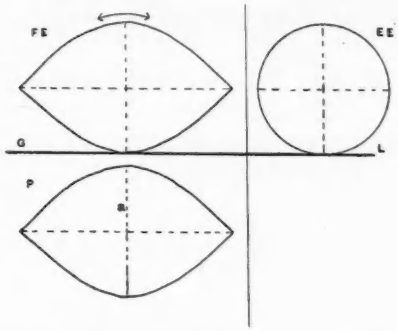


FIG. 5. Cosine spindle rocking on a horizontal plane. *GL* is the ground line and *P*, *FE*, *EE* are the plan, front elevation, and end elevation, respectively. Curved arrows indicate the direction of rocking.

The expressions for k^2 and L given for the oblate spheroid hold in this case also. The prolate spheroid was also made in the same way as the oblate spheroid. Its dimensions are $a=17.55$ cm, $b=c=14.15$ cm, whence L(calc.) $=20.0$ cm, and T(calc.) $=0.89$ sec.

The intervals for 10 rocks were measured, the spheroid being turned over from time to time. The stop-watch read 9.0, 7.0, 6.8, 9.8, 7.2, 9.3, 9.1, 8.7, 9.2, 8.7, 7.4, 8.8 sec. No reading has been rejected. Mean T(obs.) $=0.84$ sec. Rejection of the second and third observations raises T(obs.) to 0.88 sec.

The period of rocking of an ellipsoid of unequal semi-axes a, b, c is given as an exercise in Lamb.[6]

6. ROCKING OF A COSINE-SPINDLE

Let the curve $y=a\cos(x/b)$ rotate around the x axis. For large values of the angle the whole figure traced out will be a linkage of spindles, but if (x/b) is limited to the range $-\pi/2$ to $+\pi/2$, a single link is obtained. This might be called a *cosine spindle;* an oblate cosine spindle if its x dimension is less than its y dimension, and a prolate cosine spindle if vice versa. In my case I have taken a spindle shaped as in Fig. 5. Such a spindle rocks on the table if set in vibration about a horizontal y axis.

[6] See Reference 2, Ex. XV, p. 212, No. 14.

The volume of the spindle is given by

$$V=2\int_0^{b\pi/2}\pi(a\cos x/b)^2dx=\pi^2a^2b/2.$$

The second moment of the spindle about the axis of rocking is given by

$$I=2\int_0^{b\pi/2}\pi(a\cos x/b)^2[(a\cos x/b)^2/4+x^2]dx$$

$$=\pi^2\frac{3}{32}a^4b+\pi^4\frac{a^2b^3}{24}-\frac{\pi^2a^2b^3}{4}.$$

The square of the required radius of gyration is given by $k^2=I/V$ and once R is found, L may be calculated from Eq. (3).

The radius of curvature of the curve $y=a\cos x/b$ is given by

$$R=(b^2/y)/[1+(a^2/b^2)\sin^2x/b]^{\frac{1}{2}}.$$

In particular R at the end of a y axis $=b^2/a$.

The workshop turned on the lathe a solid wooden spindle after the manner described above for the spheroids. It was made for a geometry class and not for this particular experiment or it might have been made with greater accuracy.

Its dimensions are a and b each 4.00 cm so that the length of the spindle is $4\pi=12.56$ cm and the diameter $=8.00$ cm. Its volume is 316 cm^3 (also checked by water-displacement). Its second moment about a y axis $=2577$ cm^5, whence $k^2=8.16$ cm^2. (A torsion pendulum check, however, gave $k^2=8.47$ cm^2; the difference must be due to a slight error of shape.)

The experimental determination of the period of rocking gave variable results. The angular amplitude was usually about 10° which corresponds to an up-and-down amplitude of the tips of the spindle of about 1 cm, but it varied much during the period of timing. The intervals for 10 rocks were 7.2, 6.2, 6.9, 7.9, 6.8, 7.1, 7.1, 7.5, 7.2 sec yielding $T=0.72$ sec. After a year a repetition gave 6.8, 7.3, 6.9, 7.2, 6.1, 7.3, yielding T(obs.) $=0.69$ sec. Thus we can say T(obs.) $=0.7$ sec.

But how can one calculate T? The value of L calculated by the standard expression is $(8.2+4^2)/(4-4)=\infty$, or the spindle is in neutral equilibrium when resting in its normal position.

Calculation shows that at surface points on the spindle of x values 1, 1½, 2 cm the values of R of the profile are 4.5, 5.2, 6.2 cm, respectively, and the distances of these points from the center of the spheroid are all very near 4.0 cm. Substituting in the usual expressions gives periods of 1.4, 0.9, 0.7 cm, respectively. Therefore, if we allow for an amplitude of rocking giving a 2-cm variation in the x amplitude of the point of contact, some kind of agreement between theory and practice is obtained. The central portion of the spindle used was nearly spherical in shape and a plane section of the spindle near the ends of a y axis is very nearly a sphere. This was also verified by holding the long axis of the spindle horizontal and nearly submerging the spindle in water when it was seen that the unsubmerged portion was very nearly circular in outline.

Possibly owing to imperfections in the manufacture the spindle had a period of very nearly 0.5 sec for very small amplitudes (1 mm up-and-down movements of the end points) and a period of very nearly 1.1 sec for amplitudes as large as possible. The experiment should be repeated with accurate spindles of different shapes—long and short.

7. ROCKING OF A BALL RESTING IN AN ELLIPTIC HOLE

Lamb gives this problem.[7] An elliptic hole of semi-axes a, b is cut in a thin horizontal sheet of metal and a sphere of radius $c(c>b)$ rests on it, its center being therefore at a height $h=(c^2-b^2)^{\frac{1}{2}}$. If the sphere be set rocking through a small angle, the length of the equivalent simple pendulum is given by

$$L=\frac{k^2+h^2}{h}\cdot\frac{a^2-b^2}{b^2}, \qquad (6)$$

where k is the radius of gyration of the sphere about a diameter.

Figure 6 shows in plan and elevation the sphere in its "rest" position in the elliptic hole, the center of the sphere (C in plan, C_1 in elevation) being at a height h above the plane of the hole; and on the right-hand side the sphere in the extreme position of its vibration, the

[7] H. Lamb, *Higher Mechanics* (Cambridge University Press, London, 1929), p. 253, Ex. XIV, No. 5.

center being at a height $h_1(h_1>h)$. Points O and O' are the plan and elevation, respectively, of the center of the sphere in its extreme position.

The sphere is touching the ellipse at P and Q, where OP, OQ are normals to the ellipse from O. We see that $h_1^2+OP^2=c^2$, where $c=$ the radius of the sphere. Hence, to get h, it is necessary to express OP in terms of X, where X is the distance of O from C, the center of the ellipse. Let the coordinates of P be x_1y_1.

The gradient (or slope) of the tangent to the ellipse $x^2/a^2+y^2/b^2=1$ at x_1y_1 is $-b^2x_1/a^2y_1$. The gradient of the normal at x_1y_1 is, therefore, a^2y_1/b^2x_1.

The equation of OP is $(y_1-0)/(x_1-X)=a^2y_1/b^2x_1$ or $x_1-X=b^2x_1/a^2$.

Let $e=$eccentricity of the ellipse. Then $e^2=(a^2-b^2)/a^2$ or $b^2/a^2=1-e^2$. Therefore

$$x_1-X=(1-e^2)x_1 \quad \text{or} \quad x_1=X/e^2$$

and

$$(x_1-X)=X(1/e^2-1).$$

Now

$$y_1^2=b^2(1-x_1^2/a^2)=b^2(1-X^2/e^4a^2)$$
$$=b^2-X^2(1-e^2)/e^4.$$

Also

$$OP^2=OT^2+TP^2=(x_1-X)^2+y_1^2$$
$$=X^2(1/e^4-2/e^2+1)+b^2-X^2(1-e^2)/e^4$$
$$=b^2-X^2(1/e^2-1).$$

But

$$h_1^2+OP^2=c^2;$$

therefore,

$$h_1^2=c^2-b^2+X^2(1/e^2-1)=h^2+X^2(1/e^2-1).$$

If X is kept small in comparison with h, we may make the approximation $h_1=h+X^2(1/e^2-1)/2h$.

So far is geometry; now for the mechanics. In this motion the potential energy of the sphere in the extreme position may be equated to the kinetic energy of the sphere in the central position.

Let $\Omega=$angular velocity of rotation of the sphere in the central position. Then $mg(h_1-h)=\frac{1}{2}(k^2+h^2)\Omega^2$. If $v=$linear velocity of the center of the sphere in the central position, $v=h\Omega$. We see that the motion is simple harmonic of amplitude X. Remembering that the velocity of the vibrating point at the central position of the simple harmonic motion is equal to the speed

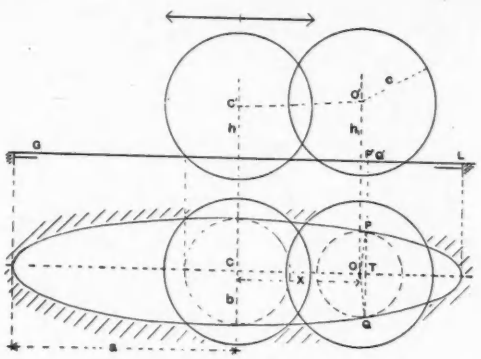


FIG. 6. Ball rocking in an elliptic hole. Plan and elevation are shown with the ball in two positions, one central and the other when the ball is displaced a distance X. Arrows indicate direction of rocking.

of the reference point traveling around the auxiliary circle or circle of reference, we get $h\Omega=2\pi X/T$, where $T=$the period of vibration.

The energy equation now becomes

$$mgX^2(1/e^2-1)/2h=\tfrac{1}{2}(k^2+h^2)4\pi^2X^2/T^2h^2,$$

or

$$T^2=4\pi^2(k^2+h^2)/gh(1/e^2-1).$$

Therefore, L, the length of the equivalent simple pendulum, is

$$(k^2+h^2)/h(1/e^2-1)=[(k^2+h^2)/h][(a^2-b^2)/b^2],$$

which is Lamb's expression Eq.(6).

A proof using moments and angular acceleration may also be obtained in this as in our other problems.

The easiest way to get elliptic holes is to get the work-shop to cut brass pipes obliquely, and to polish the sections. In the experiment the pipe must be held in a firm clamp and the horizontality of the elliptic hole tested with a spirit level.

First Trial

A large steel (S.K.F.) ball bearing was used. Dimensions: of ball, $2c=6.34$ cm; of ellipse, $2a=6.80$ cm and $2b=4.80$ cm. Therefore $k^2=2c^2/5=4.02$ cm^2, $h^2=4.29$ cm^2, $h=2.07$ cm;

$$L(\text{calc.})=\frac{4.02+4.29}{2.07}\cdot\frac{5.80\times1.00}{5.76}=4.05 \text{ cm};$$

$$T(\text{calc.})=(\tfrac{1}{5})(4.05)^{\frac{1}{2}}=0.40 \text{ sec.}$$

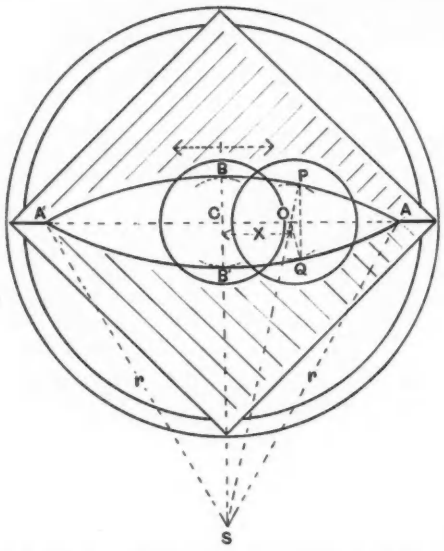


FIG. 7. Ball rocking in a hole formed by circular arcs. Plan showing ball in central and a displaced position. Arrows indicate direction of rocking.

The "rocking" was timed with a stop-watch reading to tenths of a second. Five sets of 20 "rocks" were taken: Over-all intervals were 8.3, 8.3, 8.1, 8.2, 8.3 sec. Mean = 8.24 and therefore T(obs.) = 0.41 sec—a fairly good agreement with theory.

Second Trial

A large hollow india rubber ball, hardened with age, was rocked on the same elliptic hole, $2c$ (external) = 11.3 cm. Therefore, $k^2 = \frac{2}{3}(11.3/2)^2 = 21.3$ cm², $h^2 = 26.2$ cm², $h = 5.11$ cm. Hence

$$L(\text{calc.}) = \frac{47.5}{5.11} \cdot \frac{5.80}{5.76} = 9.3;$$

$$T(\text{calc.}) = \tfrac{1}{5}(9.3)^{\frac{1}{2}} = 0.61 \text{ sec.}$$

This ball did not rock many times (new rubber balls hardly rock at all). The intervals for 5 rocks were 3.2 and 3.1 secs, whence T(obs.) = 0.63 sec.

Third Trial

A ping-pong ball was rocked on a smaller ellipse. Dimensions: of ball, $2c$ (external) = 3.78 cm; of ellipse, $2a = 3.1$ cm, $2b = 2.2$ cm. Therefore, $k^2 = \frac{2}{3}(1.89)^2 = 2.38$ cm², $h^2 = 2.36$ cm², or $h = 1.54$ cm.

$$L(\text{calc.}) = \frac{4.74}{1.54} \cdot \frac{1.19}{1.21} = 2.82;$$

$$T(\text{calc.}) = \tfrac{1}{5}1.68 = 0.34 \text{ sec.}$$

Thirty rocks could be timed, mean interval 10.5 sec, whence T(obs.) = 0.35 sec.

Fourth Trial

The same large steel ball was used as in first trial with the same pipe as in first trial but cut more obliquely. Dimensions of ellipse: $2a = 13.65$ cm, $2b = 4.86$ cm. Therefore, $h^2 = 4.14$ cm² and $h = 2.03$ cm. Therefore,

$$L(\text{calc.}) = \frac{8.16}{2.03} \cdot \frac{40.7}{5.91} = 27.6 \text{ cm};$$

$$T(\text{calc.}) = 1.05 \text{ sec.}$$

Time intervals of 10 rocks (taken in one run as amplitude diminished from 3 cm to 3 mm) were 10.85, 10.80, 10.85, 10.25, 10.10, 10.20 sec. Over-all mean was 10.6 sec. Means of the last three were 10.2 sec; whence T(obs.) = 1.05 to 1.02 sec—a fair agreement with theory.

8. ROCKING OF A BALL IN A HOLE FORMED BY TWO CIRCULAR ARCS

Figure 7 shows in plan the hole $ABA'B'$ formed between two equal arcs ABA', $AB'A'$. Denote the radius of either arc by r, AA' by $2a$, BB' by $2b$, and the radius of the ball by c. As in the last case when the center of the ball is at O (displacement = $CO = X$) the ball is touching the edges of the hole at P and Q, where OP and OQ are normal to the edges. Therefore, with a circular arc PO passes through the center S of the arc ABA'. It now remains to find h and h_1, the heights of the center of the ball above the plane of the hole when the ball is in the central and displaced positions, respectively. Clearly

$$h = (c^2 - b^2)^{\frac{1}{2}}, \quad \text{and} \quad h_1 = (c^2 - OP^2)^{\frac{1}{2}}.$$

Now $SO^2 = (r-b)^2 + X^2$. Hence, $SO = r - b + X^2/[2(r-b)]$, if $X/(R-b)$ is small in comparison with unity. Therefore

$$OP = b - X^2/[2(r-b)],$$

$$h_1^2 = c^2 - b^2 + bX^2/(r-b)$$

$$= h^2 + bX^2/(r-b),$$

and

$$h_1 = h + bX^2/[2h(r-b)].$$

Let us call the X position the extreme position, i.e., X = the amplitude. We now equate the loss of potential energy of the ball as it rolls from the extreme position to the central position to the kinetic energy of the ball in the central position and get as before

$$mg(h_1 - h) = \tfrac{1}{2}m(k^2+h^2)\Omega^2.$$

This becomes

$$mgbX^2/[2h(r-b)] = \tfrac{1}{2}m(k^2+h^2)\frac{4\pi^2}{T^2}\frac{X^2}{h^2},$$

whence

$$T^2 = \frac{4\pi^2}{g}\frac{k^2+h^2}{h}\frac{r-b}{b},$$

or

$$L = [(k^2+h^2)/h][(r-b)/b]. \qquad (7)$$

On the junk shelves of the workshop was found a large brass plate about 28 cm square and 2 mm thick from which a large disk had been cut leaving a circular edge of diameter 27.7 cm. The workman now cut from this two portions having equal chord lengths of about 12 cm, keeping sufficient backing of plate to preserve rigidity, and soldered these portions in position, to form the required hole, upon the upturned edge of a short length of thick brass piping (14-cm o. d. and 3 cm thick). This thick pipe serves as a heavy rigid support and elevates the hole sufficiently that the rolling ball is free of the table. The final appearance is shown in Fig. 7.

First Trial

A large steel ball (S.K.F. $2\frac{1}{2}$-in. diam.) is used; $c = 3.17$ cm. The hole dimensions are $r = 13.85$ cm, $2a = 12.3$ cm, $2b = 2.86$ cm. From these we compute $k^2 = 4.02$ cm^2, $h^2 = 8.01$ cm^2, $h = 2.83$ cm and obtain

$$L = \frac{4.02+8.01}{2.83}\frac{13.85-1.43}{1.43} = 37.0 \text{ cm},$$

Therefore,

$$(L)^{\frac{1}{2}} = 6.08, \quad \text{and} \quad T = 1.22 \text{ sec.}$$

Intervals for 10 rocks were observed almost consecutively as the amplitude varied from about 2 cm at the start to 2 mm at the end. Results: Run 1, 13.3, 13.1, 13.0, 12.7, 12.4, 12.5, 12.4, 12.1, 12.3, 12.1, 12.4; Run 2, 14.0, 13.5, 13.2, 13.05, 12.7, 12.4, 12.35, 12.3, 12.35, 12.1. It is seen here how the period varies with the amplitude, and that for small amplitudes the period is constant. T(obs.) = 1.23 sec.

Second Trial

A ping-pong ball is used in a different hole. The dimensions are $2c = 3.78$ cm, $r = 13.85$ cm, $2a = 13.85$ cm, $2b = 3.72$ cm, $c^2 = 3.57$ cm^2, $k^2 = 2.38$ cm^2, $h^2 = 0.11$, and $h = 0.33$ cm. Hence

$$L = \frac{2.49}{0.33}\frac{12.0}{1.86} = 48, \quad L = 6.9 \text{ cm},$$

and

$$T(\text{calc.}) = 1.4 \text{ sec.}$$

The ball did not rock many times, and its sinking at the center was very noticeable (1.89 is not much greater than 1.86). Sets of three rocks were timed. Results: 4.3, 4.3, 4.5, 4.2 sec. Mean T(obs) = 1.4 sec.

It is interesting to note at this point that the factor $(r-b)/b$ in Eq. (7) for the hole with circular arcs, if applied to the elliptic hole at the end of its minor axis becomes $(a^2/b-b)/b$ or $(a^2-b^2)/b^2$, which is identical with the shape factor in Eq. (6).

9. ROCKING OF A BALL IN A DOUBLE V-SHAPED GROOVE BETWEEN FOUR STRAIGHT EDGES

The apparatus is as shown in Fig. 8. The four rods are of polished brass one foot long and $\frac{1}{2}$ in. square. The separation at the center is adjustable and was 4.0 cm in the experiment detailed below.

Theory and practice show that the motion is not simple harmonic (an X term as well as an X^2 term enters into the value of (h_1-h). In the experiment there is also a very definite jolt as the ball passes through the central position where

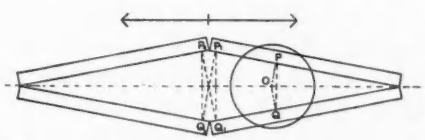


FIG. 8. Ball rolling in a long narrow V-groove formed by four straight equal rods. Plan showing central and a displaced position. Arrows indicate direction of rocking.

the instantaneous axis of rotation changes suddenly from P_1Q_1 to P_2Q_2.

With a steel ball of diameter 4.76 cm a single vibration starting with an X distance of 9 in. finished up with $X=6$ in., the time taken being about $5\frac{1}{2}$ sec; another starting at $X=6$ in. finished up with $X=4$ in. and took about 4 sec. The arrangement is, perhaps, not worth further study at this stage.

10. THE ROCKING OF A DOUBLE-ENDED CONE IN A HOLE FORMED BY CIRCULAR ARCS AND IN ELLIPTICAL HOLES

The workshop made a brass double-ended cone of total length 6.28 cm and of diameter 2.54 cm. This was now rocked in the different openings mentioned above.

10(a) Cone Rocking in the Hole Formed by Circular Arcs

The diagram (Fig. 9) shows the set-up for the rocking of this cone in the hole (the same as previously used, see Fig. 7) formed by circular arcs. If placed symmetrically across the hole and set in motion it rolls to and fro along the hole, and it is shown below that the period is equal to that of a simple pendulum of length

$$L=\frac{k^2+h^2}{h^2}\frac{rl}{c}. \tag{8}$$

k and h have been defined previously; r, l, c are, respectively, the radius of the arc of the hole, the half-length, and half-diameter of the double cone.

The diagram (Fig. 9) shows, in the middle, a plan of the hole with the cone in two positions, one when central and the other when the cone is displaced. The upper part of the figure is a front elevation. The two lower insets show, on the left, a side elevation of the cone when in the central position, and on the right, a side elevation of the cone in a displaced position. The lettering is similar to that of Figs. 6 and 7.

The displacement of the cone CO is equal to X; the heights of the center of the cone above the plane of the hole are h and h_1 in the central and displaced positions, respectively. The cone rests on the guiding arcs at P and Q which are nearly on the lowest generating lines of the cone, and POQ is at right angles to AA'.[8] Line POQ meets a parallel to AA' through S in N. The ordinate of P is y, the lettering of the two insets is obvious.

Proof of the expression for L. Let the maximum half-width of the hole be $BC=b$. From the two inset diagrams we see $h/c=(l-b)/l$, $h_1/c=(l-y)/l$, and therefore $h_1-h=c(b-y)/l$. In the triangle SPN

$$PN^2+SN^2=SP^2.$$

Therefore,

$$(OP+ON)^2+SN^2=SP^2,$$

$$(y+r-b)^2+X^2=r^2,$$

and

$$y+r-b=(r^2-X^2)^{\frac{1}{2}}=r(1-X^2/2r^2)=r-X^2/2r$$

nearly, for X/r may be made small. From this it follows that $b-y=X^2/2r$ and

$$h_1-h=c(X^2/2r)/l.$$

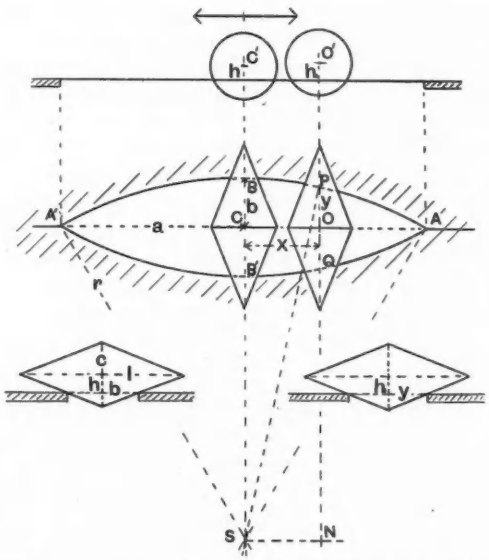


FIG. 9. Double-ended cone rocking in a hole formed by two equal circular arcs showing plan, front elevation, and two side elevations (as insets). Arrows indicate direction of rocking.

[8] Strictly speaking the plane of the hole cuts the cone in a hyperbola, and it is this hyperbola which determines the points of contact. The hyperbola is, in our case, sharp-tipped and the points of contact are very near the points P and Q on the line POQ, though really a little to the right of P and Q.

The mechanics of the problem is as in previous sections and the expression $mg(h_1-h) = \frac{1}{2}m(k^2+h^2)\Omega^2$ yields $T=(4\pi^2/g)((k^2+h^2)/h^2)(rl/c)$; or L, the length of the equivalent simple pendulum, is equal to $((k^2+h^2)/h^2)(rl/c)$.

In the experiment with the given cone and given hole $l=3.14$ cm, $c=1.27$ cm, and $r=13.85$ cm. Therefore $h=1.27(3.14-1.86)/3.14=0.518$ cm, whence $h^2=0.268$ cm^2. For a cone the square of the radius of gyration about the central axis is given by $k^2=(3/10)c^2$ and therefore $k^2=(3/10)(1.27)^2=0.484$ cm^2. Hence

$$L=\frac{0.484+0.268}{0.268}\,\frac{13.85\times 3.14}{1.27}=96.1 \text{ cm},$$

and $T=\frac{1}{5}L^{\frac{1}{2}}=\frac{1}{5}(9.80)=1.96$ seconds. The cone rolled to-and-fro very nicely in the hole, and the measured intervals of five vibrations were 9.4, 9.6, 9.6, 9.6, 9.7 seconds, respectively. Mean T(obs.) was 1.92 sec. The agreement is good.

10(b) Cone Rocking in an Elliptic Hole

The cone was now rocked in an elliptic hole (refer to Fig. 6). In this case, again denoting OP by y we have $y=b(1-X^2/a^2)^{\frac{1}{2}}=b(1-X^2/2a^2)$ nearly, and $h_1-h=c(b-y)/l=cbX^2/2la^2$. Therefore

$$L=((k^2+h^2)/h^2)((la^2)/(cb)). \qquad (9)$$

First Trial

The cone was rocked on an elliptic hole of $2a=13.65$ cm and $2b=4.85$ cm. In this case $h=1.27(3.14-2.43)/3.14=0.287$ and $h^2=0.0825$ cm^2; also

$$k^2+h^2=0.484+0.082,$$

and so

$$L=\left(\frac{0.566}{0.0825}\right)\left(\frac{3.14\times 6.82^2}{1.27\times 2.43}\right)=325 \text{ cm},$$

whence

$$T(\text{calc.})=\tfrac{1}{5}L^{\frac{1}{2}}=\tfrac{1}{5}(18.0)=3.60 \text{ sec}.$$

The cone did not make many complete vibrations and the intervals occupied by just two complete vibrations were measured. They are 7.4, 6.7, 6.2, 7.4, 6.0, 7.0, 7.4, 7.1, 6.0, 7.2, the vibrations of larger amplitudes giving the shorter periods. The mean is 6.84 or T(obs.) $=3.42$ sec.

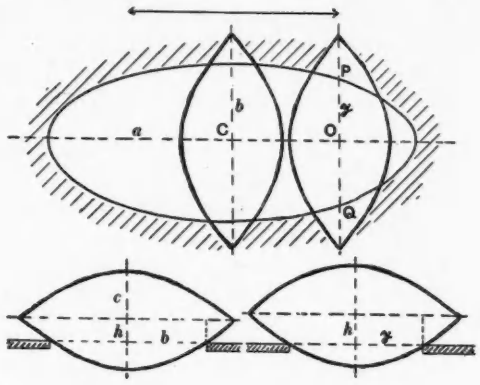


FIG. 10. Cosine spindle rocking in an elliptic hole. The arrows indicate the direction of rocking. The lower inserts are side views, the left one when the spindle is in the central position and the right one when the spindle is in the displaced position.

Second Trial

The cone was rocked on a much smaller ellipse. It rocked many times. Here $2a=2.60$ cm and $2b=2.22$ cm.

In this case $h=(1.27/3.14)(3.14-1.11)=0.821$ cm and $h^2=0.674$ cm^2.

$$L=\frac{(0.484+0.674)}{0.674}\,\frac{3.14\times 1.30^2}{1.27\times 1.11}=6.47 \text{ cm}$$

and $T(\text{calc.})=\frac{1}{5}(6.47)^{\frac{1}{2}}=\frac{1}{5}\times 2.54=0.51$ sec.

Observed intervals for 10 vibrations of very small amplitude are 5.0, 5.0, 5.1, 5.1, 5.0, 5.1, 4.9, 5.0 sec; mean 5.0 sec, whence T(obs.) $=0.50$ sec. The agreement is good and is also good for the other ellipses tried.

10(c) Cone Rocking in the Long V-Shaped Groove

As with a ball the motion is not simple harmonic, and no results are given.

11. THE COSINE SPINDLE ROLLING TO-AND-FRO IN AN ELLIPTIC HOLE

The rolling is in the direction at right angles to the axis of revolution of the spindle and in the direction of the longer axis of the ellipse.

The arrangement is shown in plan and end-view elevation in Fig. 10. We shall assume that contact between spindle and hole occur at points P and Q, where POQ is at right angles to the

longer axis of the ellipse. Strictly speaking the points of contact are just to the right of P and Q. The semi-axes of the ellipse are a and b. To avoid confusion we shall write the equation of the profile of the spindle as $y=c\cos x/d$.

With the same nomenclature as before the height h of the center of the spindle when in the middle of its vibration is equal to $c\cos b/d$, and the height h_1 in the displaced position X is given by $h_1=c\cos y/d$, where the relation between y and X is given by $X^2/a^2+y^2/b^2=1$. Therefore

$$\begin{aligned} h_1-h &= c[\cos(y/d)-\cos(b/d)] \\ &= [\cos(b/(1-X^2/a^2)^{\frac{1}{2}}/d-\cos(b/d)] \\ &= c[\cos(b/d-bX^2/2a^2d)-\cos(b/d)], \end{aligned}$$

and if X^2/a^2 is small,

$$\begin{aligned} h_1-h = c[&\cos(b/d)\cos(bX^2/2a^2d) \\ &+\sin(b/d)\sin(bX^2/2a^2d)-\cos(b/d)]. \end{aligned}$$

As $bX^2/2a^2d$ is a small angle its cosine may be replaced by unity and its sine by the angle itself. Therefore,

$$\begin{aligned} h_1-h &= c[\cos(b/d)+(bX^2/2a^2d)\sin b/d-\cos(b/d)] \\ &= (bcX^2/2a^2d)\sin(b/d). \end{aligned}$$

So far is geometry. Introducing the mechanics equation, with nomenclature as before, we write

$$\begin{aligned} mg(bcX^2/2a^2d)\sin(b/d) &= \tfrac{1}{2}m(k^2+h^2)\Omega^2 \\ &= \tfrac{1}{2}m(k^2+h^2)\frac{4\pi^2}{T^2}\frac{X^2}{h^2}. \end{aligned}$$

Therefore,

$$L=\frac{k^2+h^2}{h^2}\,\frac{a^2d}{bc\sin(b/d)}. \tag{10}$$

In one experiment an elliptic hole of dimensions $a=6.83$ cm and $b=2.43$ cm was used. The same spindle was used as described earlier. Its dimensions are given by $c=d=4.00$ cm and $k^2=6.00$ cm^2 (obtained from the first term in the expression for I in sec. 6). The value of $b/d=0.607$ radian and the cosine and sine of this angle are 0.822 and 0.571, respectively, so that $h=3.29$ cm. Therefore,

$$L(\text{calc.})=\frac{6.00+3.29^2}{3.29}\,\frac{(6.83)^2\times4.00}{2.43\times4.00\times0.571}=52,$$

and $T(\text{calc.})=\frac{1}{5}(52)^{\frac{1}{2}}=\frac{1}{5}\times7.2=1.4$ sec.

Direct determination of T showed little change of T with amplitude. A number of intervals for 5 rocks gave the values 7.1, 6.9, 7.0, 6.8, 6.8, 7.0, 6.8, 6.8, 6.8, 7.2, 7.0 with a mean value of $T(\text{obs.})=1.39$ sec, a fair agreement. If the axis of the spindle is turned to be parallel to the longer axis of the ellipse the period of rocking changes to about 0.65–0.75 sec for moderately large amplitudes and to about 0.4 sec for very small amplitudes. These values are not very different from those obtained when the spindle rocked on the table.

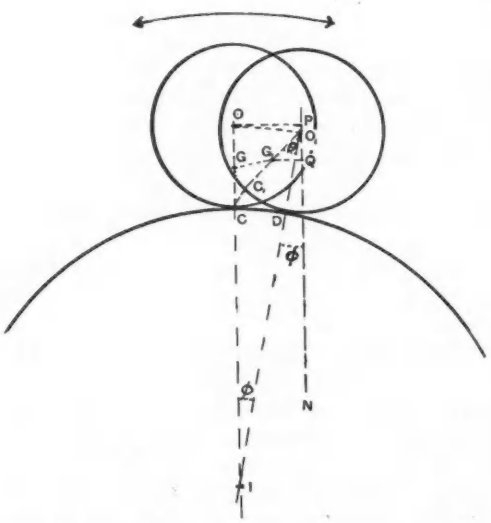


FIG. 11. Cylinder resting on and rocking on the top of a fixed cylinder. Arrows indicate direction of rocking.

12. A CYLINDER ROCKING UPON THE TOP OF A FIXED CYLINDER (BOTH AXES HORIZONTAL AND PARALLEL)

Lamb[9] also considers the rocking of an eccentrically loaded cylinder resting on the top of a larger fixed cylinder. The lower cylinder (of radius c) has its axis at I (Fig. 11). The upper cylinder (of radius a, where $a<c$) is loaded eccentrically so that its mass-center G is below its geometric-center O by an amount b. In the normal position of rest I, O, G, and the point of contact C are in the same vertical. Let the upper

[9] H. Lamb, see reference 2, p. 190. See also M. Scott, *Mechanics, Statics, Dynamics* (McGraw-Hill Book Company Inc., New York, 1949), pp. 346–349.

cylinder roll without slipping slightly to the right to a new position where D is the point of contact and O_1, G_1 are the new positions of O and G. The normal point of contact C of the upper cylinder has moved up to C_1 and, if the angles CID, C_1O_1D are denoted by ϕ and θ, respectively, we have $c\phi = a\theta$. If G_1 is at a higher elevation than G the equilibrium is stable and the upper cylinder will rock to and fro.

To find the change of potential energy between the normal and displaced positions draw a vertical PO_1QN through O_1 and horizontals OP, G_1Q to meet this vertical in P and Q, respectively. In the motion back to the central position the geometric center O rises a distance $O_1P = (c+a)(1-\cos\phi)$ and, relative to the geometric center, the mass-center G falls a distance $OG_1 - O_1Q = b\{1-\cos(\phi+\theta)\}$. Hence the loss of potential energy $= mg[b\{1-\cos(\phi+\theta)\} - (c+a)(1-\cos\phi)]$, where m = mass of the upper cylinder. The angles ϕ and θ are supposedly very small so that their cosines may be replaced by the first two terms in their expansions; also $\theta = c\phi/a$, and therefore, the loss of potential energy from the displaced position to the central position is

$$mg[b\{(c+a)/a\}^2 - (c+a)]\phi^2/2.$$

This loss of potential energy must be equated to the kinetic energy in the central position. Therefore, if Ω is the angular velocity of rotation of the upper cylinder about the point of contact C this kinetic energy is $\frac{1}{2}m(k^2+GC^2)\Omega^2$. Distance GC may be written either as h or $(a-b)$. Replace Ω by $\{2\pi(c+a)\phi\}/Ta$ as in earlier problems and we get the equation

$$mg[b\{(c+a)/a\}^2 - (c+a)]\phi^2/2 = \tfrac{1}{2}m\{k^2+(a-b)^2\}\{2\pi(c+a)\phi\}^2/T^2a^2,$$

whence we see that the motion is simple harmonic and the length L of the equivalent simple pendulum is given by

$$L = \frac{\{k^2+(a-b)^2\}\{(c+a)/a\}^2}{b\{(c+a)/a\}^2-(c+a)} = \frac{k^2+(a-b)^2}{b-a^2/(c+a)}, \qquad (11a)$$

or by its equivalent expression in terms of h:

$$L = (k^2+h^2)/\{(ac)/(c+a)-h\}. \qquad (11b)$$

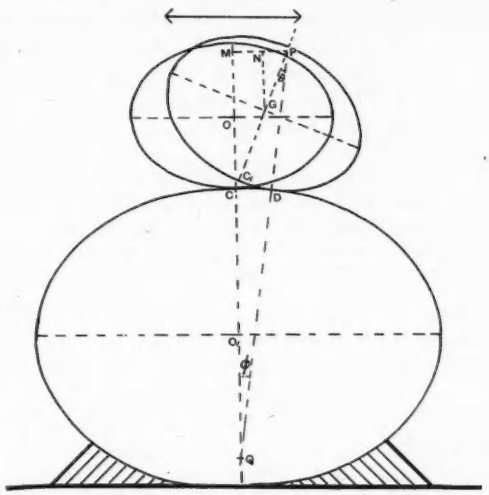


FIG. 12. Elliptic band resting on and rocking on the top of a fixed ellipse. Arrows indicate the direction of rocking.

In an experiment the fixed cylinder was an old pulley wheel which was found on the junk shelves of the workshop. The rolling cylinder was the ring of Fig. 1(a) loaded now with a heavier brass cylinder in order to increase the value of b. I omit some details. The pertinent dimensions were $c = 17.0$ cm, $a = 5.00$ cm, $b = 1.27(0)$ cm, and $k^2 = 5.89$ cm^2. Hence

$$L = (5.89+(5.00-1.27)^2/(1.270-1.136) = 148,$$

whence $L = 12.2$ cm and T(calc.) $= 2.43$ sec.

It was not possible to observe more than about four vibrations. Several sets of three vibrations were timed. The results were 7.7, 7.4, 7.7, 7.1, 7.3, 7.5, 7.5, 7.4 sec. Hence T(obs.) $= 2.5$ sec, a fair agreement.

Scott points out that Eqs. (11a) and (11b) may be applied to: (1) The case where the lower cylinder has its radius so increased that it becomes a horizontal table, i.e., $c = \infty$. Equation (11a) now becomes $L = \{k^2+(a-b)^2\}/b$ as in Eq. (1). (2) The case where the lower cylinder becomes a concave surface and the upper cylinder a rolling ball (a familiar experiment described in many books). Replace c by $-c$ and Eq. (11b) now becomes

$$L = (k^2+a^2)\Big/\left(\frac{ac}{c-a}-a\right) = \frac{7}{5}(c-a).$$

(3) The case where a becomes infinite so that the upper cylinder is replaced by the teetering rod of the second experiment of this article; Eq. (11b) now becomes $L=(k^2+d^2)/(c-d)$, where the c here is the a of Eq. (4).

13. AN ELLIPTIC BAND RESTING AND ROCKING ON THE TOP OF A FIXED LARGER ELLIPSE, THE MAJOR AND MINOR AXES OF BOTH ELLIPSES BEING HORIZONTAL AND VERTICAL, RESPECTIVELY

Figure 12 shows the arrangement. Let a_1, b_1 and a_2, b_2 be the semi-axes of the rocking and fixed ellipses, respectively, and R_1, R_2 be the radii of curvature of the two ellipses at the ends of the minor axes. The center of gravity of the upper ellipse is at its geometric center O. In the central position C is the point of contact between the two ellipses. When the upper ellipse is slightly displaced, the point of contact moves to D and the original point of contact of the upper ellipse moves to C_1 and the center of gravity up to G. The centers of curvature of C_1D and CD are at P and Q both on the minor axes of their respective ellipses, and $PDQP$ is a straight line. Denote the angles CQD, C_1PD by ϕ and θ, respectively, and we see that since $CD=C_1D$, $R_2\phi=R_1\theta$. To find the increase in elevation of the center of gravity of the upper ellipse draw PM horizontally to cut the vertical QCO in M and draw GN vertically to cut PS in N. In the central position the height of the center of gravity of the upper ellipse above Q is R_2+b_1. In the displaced position the height of GN_1 above Q is

$$\begin{aligned} QM-GN_1 &= (R_1+R_2)\cos\phi-(R_1-b_1)\cos(\phi+\theta) \\ &= (R_1+R_2)(1-\phi^2/2) \\ &\quad -(R_1-b_1)(1-(\phi+\theta)^2/2). \end{aligned}$$

Therefore, the height of G above O is $(R_1-b_1)(\phi+\theta)^2/2-(R_1+R_2)\phi^2/2$, and this on replacing θ by its value $R_2\phi/R_1$ reduces to

$$\{(R_1-b_1)[(R_2+R_1)/R_1]^2-(R_1+R_2)\}\phi^2/2.$$

As before, the mechanics equation equating the loss of potential energy to the gain of kinetic energy as the ellipse moves back from the displaced position to the central position is

$$mg\{(R_1-b_1)[(R_2+R_1)/R_1]^2 -(R_1+R_2)\}\phi^2/2=\tfrac{1}{2}m(k^2+b_1^2)\Omega^2,$$

where Ω = angular velocity of rotation of the rocking ellipse as it passes through the central position.

Regard P as a typical point executing the simple harmonic motion of period T. If v be its velocity as it passes through the central position, $v=R_1\Omega$. The amplitude of vibration P is $(R_1+R_2)\phi$ and as v is also the constant speed of the imaginary point traveling around the circle of reference whose radius is $(R_1+R_2)\phi$ in the period T, we have

$$\Omega=v/R_1=[2\pi(R_1+R_2)\phi/T]/R_1.$$

Substitute this expression for Ω in the mechanics equation above and eventually we see that L, the length of the equivalent simple pendulum, is given by

$$L=(k^2+b_1^2)/[(R_1-b_1)-R_1^2/(R_1+R_2)]. \quad (12)$$

Note 1. If $R_2=\infty$, i.e., the lower surface is a plane surface,

$$L=(k^2+b_1^2)/(R_1-b_1),$$

which is Eq. (3).

Note 2. If the upper ellipse is loaded so that the combined center of gravity is along the minor axis at a distance b from the geometric center, the same reasoning as above leads to

$$L=[k^2+(b_1-b)^2]/ \{[R_1-(b_1-b)]-R_1^2/(R_1+R_2)\}, \quad (13)$$

and this equation naturally includes the solution of the loaded circle rocking on the top of a fixed circle [Eq. (11a)].

First Trial

The upper ellipse was unloaded, both major axes horizontal. The lower ellipse was a stout elliptic band (fitted within by its former to keep it from distortion) of outer dimensions $a_2=18.88$ cm, $b_2=12.68$ cm. The upper ellipse was an elliptic band of outer dimensions $a_1=11.65$ cm and $b_1=8.55$ cm and of thickness 2.0 mm. Hence $R_2=28.1$ cm, $R_1=15.6$ cm, $R_1^2/(R_1+R_2)=5.56$ cm, $k^2=\frac{1}{4}(11.55+8.45)^2=100.0$ cm^2 and $b_1^2=73.1$ cm^2. Therefore,

$$L=\frac{100+73}{15.60-8.55-5.56}=\frac{173}{1.49}=116 \text{ cm},$$

whence $L^{\frac{1}{2}}=10.8$ and T(calc.) $=2.16$ sec.

In experimentation the positions of the bands were reversed three times to eliminate errors in smoothness and shape. The four arrangements gave mean periods of 2.05, 2.07, 2.13, 2.28 sec, respectively, with a final mean of T(obs.) $=2.13$ sec.

Second Trial

The upper ellipse was loaded, both axes horizontal. The elliptic band was loaded with a brass cylinder resting within at the lower end of the minor axis and kept in place with a little plasticine. Mass of cylinder $=1400$ g, mass of elliptic band $=353$ g, and radius of cylinder $=2.53$ cm. Calculation shows $b=4.65$ cm, and therefore $b_1-b=3.90$ cm; also k^2 about the axis through the combined center of gravity $=28.1$ cm^2. Hence

$$L=\frac{28.1+15.2}{15.60-3.90-5.56}=7.0 \text{ cm},$$

whence $L^{\frac{1}{2}}=2.64$ and T(calc.) $=0.53$ sec. Observation gave $T=0.51$ sec and, when the upper ellipse was inverted, 0.49 sec.

Third Trial

The lower larger ellipse was now stood on end, with the upper ellipse loaded as in the second trial. R_2 now was 8.51 cm, and $R_1^2/(R_1+R_2)$ $=10.1$cm. Hence

$$L=\frac{28.1+15.2}{15.60-3.90-10.1}=27 \text{ cm},$$

whence $L^{\frac{1}{2}}=5.2$ and T(calc.) $=1.04$ sec. Observation gave $T=1.2$ sec, when one end of the big ellipse was uppermost, and 1.0 sec when the larger ellipse was reversed.

Another variation is to put the smaller elliptic band within the larger band and rocking on the lower end of its minor axis. Reasoning similar to that above gives

$$L=[k^2+(b_1-b)^2]/\{[R_1-(b_1-b)]+R_1^2/(R_2-R_1)\}, \quad (14)$$

which agrees with Eq. (13), if R_2 is replaced by $-R_2$ (its radius of curvature being now upwards). If the smaller band is unloaded b must be made zero. Equation (14) and similar equations applicable to circular bands have been confirmed by experiment. Many modifications are possible.

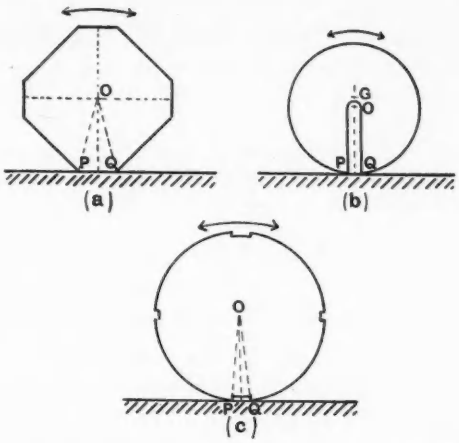


FIG. 13. (a) Rocking of a square on a bevelled off corner. (b) Rocking of a slotted weight standing on the slot. (c) Rocking of a disk standing on the edges of a shallow slot. Arrows indicate direction of rocking.

One is similar to Fig. 13 but with both ellipses free to rock. Another is to fix the lower ellipse in an inclined position and to rock on the top of it a loaded elliptic band with the load off the axis. The arithmetic becomes complicated.

14. OTHER EXAMPLES OF ROCKING

Figure 13(a) shows a square block of wood or metal standing on a corner bevelled off at 45°, Fig. 13(b) shows a slotted weight standing on the opening of the slot, and Fig. 13(c) shows a disk of metal in which four shallow depressions have been cut into the circumference. The disk is standing on the transverse ridges of one of these depressions. For small initial displacements these bodies rock alternately about the edges P and Q, and chatter as these edges are rapidly struck. There is loss of energy at each impact and the vibrations are markedly damped. The periods are small and difficult to measure, and the frequency increases as the amplitude decays like the end of the life of a coin spun on the table. A wedge of wood 5.0 inches square with a 1.0 inch triangle taken off one corner made two vibrations in about 0.8 sec, and a 100-g slotted weight of diameter 4.72 cm with a slot 2.70 cm deep and 0.73 cm wide made three rocks in about $1\frac{1}{2}$ sec.

The theory for the period of rocking in these cases may be illustrated by a consideration of the slotted disk. Suppose the disk is drawn back until

OP is vertical. On release the fall of the center of gravity from the extreme to the normal position is equal to the difference between OP and the altitude of the triangle OPQ. Let the radius of the disk $=r$ and the angle $POQ=2\theta$. The decrease of the potential energy of the disk $=mg(r-r\cos\theta)=mgr\theta^2/2$ and the mechanics equation becomes

$$mgr\theta^2/2=\tfrac{1}{2}m(k^2+r^2)\Omega^2=\tfrac{1}{2}m(k^2+r^2)(2\pi r\theta)^2/r^2T^2,$$

whence

$$L=(k^2+r^2)/r. \tag{15}$$

The value of r for the disk used was 6.0 cm hence $k^2=18.0$, $L=9.0$, and T(calc.) $=0.6$ sec. The period of rocking was determined before this calculation was made. Six rocks were timed with resulting intervals of 3.2, 2.8, 2.8, 2.9, 3.1, 3.0, 3.2, 3.4, 3.2, a mean of 3.2 sec, whence T(obs.) $=0.51$ sec. After the calculation of T was made, it was realized that the frequency rises rapidly as the amplitude decreases so the timing of only three rocks was now measured. The intervals were 1.8, 1.6, 1.6, 1.6, 1.9, 1.9, 1.9, 1.7 sec, whence T(obs.) $=0.6$ sec.

The old-established heat and sound experiment known as Trevelyan's rocker[10] is of this kind, but its vibrations are maintained by an inflow of heat energy.

The above and similar problems, the list of which could be extended, serve admirably for class use in the laboratory. The apparatus is simple and not easily damaged. Theory and practice are both interesting and after this essay in harmonic vibrations the work could be extended to cases whose rockings are not harmonic.

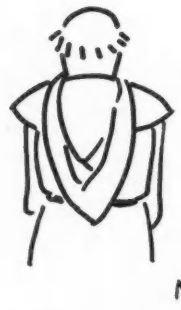
Rocking objects which occur in ordinary life experience may now be watched with interest instead of with annoyance. For example we have the distorted napkin ring rocking on the dining room table, the stack of rocking crockery, the saucepan hanging from its nail on the kitchen wall and rocking until the reproving hand brings it to rest.

[10] Rayleigh, *Theory of Sound* (Macmillan and Company, Ltd., London, 1896), Vol. II, p. 224. Barton, *Textbook of Sound* (Macmillan and Company, Ltd. London, 1898), p. 348. Sutton, *Lecture Demonstrations in Physics* (McGraw-Hill Book Company, Inc., New York, 1928), p. 202.

Comic Dictionary of Physical Terms

Illustrated by MARGE M. MULLER
East Lansing, Michigan

Transformation, classical



Transformation, canonical

Uncertainty principle

State, ground

Quantum Mechanics and Thermodynamic Continuity

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(Received March 5, 1952)

The basic concepts and rules of quantum mechanics are shown to be immediate consequences, on the basis of simple reasoning, of a postulate of continuity in the domain of macroscopic thermodynamics. The continuity principle leads first to the concept of a fractional likeness between two states in general, as opposed to the classical alternative of either like or unlike. Then all possible states of a system can be arranged in sets of mutually unlike or "orthogonal" states, with members A and B of different sets interconnected by fractional likeness factors $q(A, B)$ between zero and unity. These factors q are identical with the relative intensities in a splitting effect of a state into components, ruled by probability relations and by the basic principle of quantum statistics of not counting permutations of individual particles. The problem of relation between the various fractional likenesses, or probabilities of transition under analysis, is a purely mathematical problem and is solved by the introduction of probability amplitudes necessarily subject to a matrix law of multiplication, i.e., to the principle of superposition. Thus, starting from the continuity postulate of thermodynamics, quantum theory in its general outline is obtained by simple reasoning. All further details, such as the duality of waves and particles, the uncertainty relation, and the mechanics of special systems, need only one more bit of empirical information, namely, the symmetric and periodic form of the amplitude ψ of fractional likeness between a state of given coordinate and a state of given momentum.

1.

QUANTUM theory rests on a number of new conceptions and mathematical rules of interrelation which have proved superior to those of the classical theory. The usual approach to these new conceptions and rules is inductive. Experience concerning spectral lines and collision processes has promoted the idea of discrete energy levels together with mathematical rules of calculating their values; experiments of matter ray diffraction has brought forward the idea of a duality between waves and particles, quantitatively connected by Planck's $E=h\nu$ and de Broglie's $p=h/\lambda$, together with rules of superposition for complex amplitudes ψ; scintillation experiments have forced us to accept probability, and the thermal behavior of substances led to quantum statistics.

There is one drawback to this inductive physical approach, however: It leaves the impression that quantum theory is pieced together—as it actually was—from diverse evidence chiefly of a microphysical character. One may be wondering whether the various concepts and mathematical theorems of quantum theory might not be derivable from *one* fundamental principle of Nature representing a general type of experience and therefore acceptable without much questioning. One ought to remember in this connection that Max Planck developed his older quantum theory of 1900 not from delicate microphysical data but from the long familiar macroscopic fact of thermal stability of radiation, as contrasted to the classically expected dissipation of the radiation energy to higher and higher frequencies. In the following pages we are going to show that *general* quantum mechanics is derivable from a *postulate of continuity* for the thermal work, entropy, etc., of reaction between substances, in particular their diffusion. That a continuity principle should be responsible for a branch of physics notorious for its discontinuous jumps between discrete mechanical states is rather surprising; nevertheless, thermodynamical principles have proved much more reliable than those of mechanics which lately had to undergo two major operations in order to accommodate electrodynamics (Einstein) and energetics (Planck).

2. THE POSTULATE OF CONTINUITY

It is a well-known fact that the maximum amount of work obtainable from the isothermal diffusion of two gases depends on whether the two gases are like or unlike. When they are *like* then their diffusion does not represent a physical process at all, and work cannot be gained from it. On the other hand, when the two gases are *unlike*, or are samples of the same kind of gas in two unlike "modifications" or "states" then the maximum isothermal work obtainable from

their diffusion, called the diffusion work henceforth, has the classical value

$$W_{\text{class}}=2nRT\ln 2 \quad \text{for unlike gases,} \qquad (1)$$

when n mols of either gas diffuse from separate volumes V to the common volume $2V$ at temperature T. However,

$$W=0 \quad \text{for like gases.} \qquad (1')$$

The question now arises of what will happen to the value of W when two originally quite unlike gases, or two quite unlike states of the same kind of gas are gradually made more and more similar. Think of one gas sample consisting of molecules all pointing in one direction z, and another sample with the same kind of molecules pointing in another direction z^*, with the angle θ between z and z^* gradually decreased to zero. Is there a sudden change of W from the full value of Eq. (1) to zero of Eq. (1′) only in the last moment when the difference between the two gases changes from infinitely small to exactly zero? Or is there a gradual change from W_{class} to zero when the two diffusing gases are made more and more like? The first alternative, an abrupt change of W, would present a most paradoxical situation, known as the *Gibbs paradox*. One intuitively refuses to believe in such a discontinuity of the diffusion work upon a continuous decrease of the unlikeness between the two gas states. The root of the paradox is of course the narrow classical alternative "either like or unlike" for describing the relation between the two gas states. There ought to be a partial or fractional likeness between two gas states A and B, quantitatively defined by a factor $q=q(A,B)$ of value between 1 and 0 so as to offer a generalized diffusion work value

$$W=f(q)\cdot 2nRT\ln 2 \qquad (1'')$$

with the limiting cases $f(q)=1$ for $q=0$ (unlike gases) and $f(q)=0$ for $q=1$ (like gases). The possibility of such a gradual scale of likeness values had simply been overlooked in classical thermodynamics. The factor q of mutual likeness is symmetric in A and B. The requirement of avoiding the Gibbs paradox, namely, by a continuous scale of likeness values, may be called the postulate of thermodynamic continuity.

3. ORTHOGONAL SETS OF STATES

Important conclusions may be drawn from the continuity postulate even before discussing the quantitative aspect of the fractional likenesses between various states. Let *all* states of a certain kind of particle or system be listed on separate slips of paper forming a possibly infinitely high paper mountain. Draw one of the slips and denote the state it represents as the state A_1. Any remaining state will necessarily be either quite unlike A_1 for which we also say non-A_1, or fractionally like A_1. Next draw any one state quite unlike A_1 (non-A_1) and denote it as A_2. Next draw a state both unlike A_1 and unlike A_2 and call it A_3. Continuing in this fashion one may collect a set of mutually quite unlike or "mutually orthogonal" states $A_1A_2A_3\cdots$. When the next draw leaves no other choice than a state B_1 not orthogonal to all the previously drawn states A then the set A is "complete." Starting from B_1 we now may collect a complete set of mutually orthogonal states B, then a set C, and so forth. This procedure is not univalent, a fact known of degeneracy, and it may exhaust all possible states only after an infinite number of orthogonal sets has been collected.

The point brought out so far is this: Whereas classical theory considers any state A_1 as surrounded by an indiscriminate crowd of other states, all considered as quite *unlike* A_1, the principle of thermodynamic continuity leads to an organized arrangement of states into mutually orthogonal sets. The likeness fractions between the members of one set are

$$q(A_k,A_{k'})=\delta_{kk'}, \quad \text{and} \quad q(B_j,B_{j'})=\delta_{jj'}, \qquad (2)$$

etc., with δ being the Kronecker symbol. On the other hand, the members of different sets are connected by a web of fractional likenesses, each thread $q(A_k,B_j)$ having a definite "intensity" between 0 and 1.

4. QUANTITATIVE DETERMINATION OF LIKENESS FACTORS

If one knew the function $f(q)$ in the diffusion work formula (1″) [given later in Eq. (3)] one could use the W-measurement for finding the likeness fraction q between the two diffusing gas states. However, W can only be determined by a process of reversing the diffusion. Let us first

discuss the special case of "unfusing" the mixture of two quite unlike or orthogonal gas states, such as A_1 and non-A_1 (we use the latter expression for any state orthogonal to A_1). The separation of the gases A_1 and non-A_1 would require a sieve, or semipermeable diaphragm, or some other selective device which will pass the gas state A_1 but will block all states non-A_1. We call such a device an A_1-selector.

Now if likeness between two states means lack of separability, and unlikeness (orthogonality) means total separability, then partial or fractional likeness between the states A_k and B_j must signify a sort of partial or fractional separability—there is nothing else left besides total separability and inseparability than fractional separability. That is, the same A_k-selector which passes A_k and blocks non-A_k must be expected partially to pass and partially to repell the gas state B_j. Quantitatively, if the gas B_j is passed at the rate q_{jk} and repelled at the rate $1-q_{jk}$ by an A_k-selector, then such an experiment would define the fractional likeness q_{jk} between A_k and B_j. The postulate of thermodynamic continuity thus foreshadows a splitting effect of the state B_j into components going different "ways." Whereas the semipermeable membranes used in osmosis distinguish only between particles of a different kind, salt and solvent, selective devices distinguishing between mutually orthogonal states of the same kind of particle were first devised by Stern and Gerlach in 1921 in a planned attempt of isolating those "quantum states" which had been indirectly inferred from the analysis of optical spectra (Bohr) and collision experiments (Franck-Hertz).

We now turn to the question of *how* some B_j-particles manage to pass the A_k-selector, whereas others are repelled. Wanting to be true to our former definition of an A_k-selector (passing A_k, rejecting non-A_k) there can be only one answer: Those particles which pass must first have turned or "jumped" from the state B_j to the state A_k in order to qualify for passage, *and then stay in the new state* A_k. Similarly, those particles which are repelled must have, in contact with the A_k-selector, turned to a state orthogonal to A_k thereby forfeiting their right of passage. These sudden transitions, seen here as a necessary consequence of the continuity postulate, may be ascribed to the activity of the selector itself which challenges incoming particles B_j as to "A_k or non-A_k." The actual arrangement of Stern-Gerlach is an example of an *A-separator* which yields, instead of A_k and non-A_k on different sides, various components $A_1A_2\cdots$ all on one and the same side, yet *separately collectable* since they are going different "ways." It is not hard to imagine how by combination of various A_k-selectors ($k=1, 2, \cdots$) one can in principle construct an A-separator and vice versa, and how the latter can be transformed into an A_k-selector by reflecting the path of all rays non-A_k.

5. PROBABILITY

Another essential feature of quantum mechanics is brought out by the question of how an individual particle B_j decides whether to turn, when confronted by an A-separator, to the state A_1, whereas another particle B_j decides to turn to A_2, and so forth. Since originally there was no difference between the particles B_j, the various fates of individual particles can only be ascribed to probability, ruled by definite odds, q_{j1}, q_{j2}, $\cdots$, so that the observed relative intensities q_{jk} represent the probabilities for an individual particle to turn from B_j to A_k. Thus probability enters the scene. The sum of the relative intensities, or probabilities, is unity:

$$\sum_k q(A_k, B_j)=1, \quad \text{and} \quad \sum_j q(A_k, B_j)=1. \quad (2')$$

Some physicists have expressed the hope that a hidden difference between the particles in the original state B_j may be responsible as the cause of their different behavior. This hope is futile, and the reasoning leading to it is self-contradictory: If there were an individual cause for one B_j-particle to turn with certainty to A_1, and for another to A_2, then these particles would have to be classified as *not* originally belonging to the same state B_j but rather to different states, B_j^1 and B_j^2, which contradicts the former assumption that both particles are in the same original state B_j distinct from all other states found in the paper mountain of Sec. 3. There is no way out of the probability issue for particles as long as one acknowledges the principle of continuity and its logical consequences, the conceptions and rules of quantum theory.

6. QUANTUM STATISTICS

It is a matter of probability whether this particle B_j will turn to the state A_1 and that to A_2 when sent through an A-separator. However, when there are N particles B_j then the numbers $N_1, N_2, \cdots$ turning to $A_1, A_2, \cdots$, respectively, are dynamically *certain*, namely, $N_k = q_{jk}N$. Similarly, when exposing the same N particles to a different separator arrangement, certain occupation numbers $N_1', N_2', \cdots$ will occur with dynamical certainty again. Now it is a general principle of statistical mechanics that situations following one another with certainty are counted with the same statistical weight or *a priori* probability. In the present case, therefore, the distribution of N particles with the occupation number set $N_1N_2\cdots$ has the same weight as the original distribution of all N particles in one state B_j. The same applies to the occupation number set $N_1'N_2'\cdots$ and to any other occupation number set obtainable from the same original distribution of all N particles in one state. Hence all these distributions have the same common weight, usually counted as weight unity. We here have arrived at the basic principle of quantum statistics: Regrouping does not change the statistical weight. This is opposed to classical statistics which counts each permutation of identical particles as a separate case and assigns to the occupation number set $N_1N_2\cdots$ the weight $N!/N_1!N_2!\cdots$ and to the set $N_1'N_2'\cdots$ the weight $N!/N_1'!N_2'!\cdots$. Quantum statistics does not need a special introduction as a new principle; it rather is an immediate consequence of the splitting effect in which various resulting occupation number sets are produced with dynamical certainty. In applications of this principle a sharp distinction must be made[1] between the following two cases. (1) There are N particles and Z levels. Originally all particles B_j are in one level, the other levels being empty. Then a redistribution takes place with $N_k = q_{jk}N$ particles A_k occupying $Z_k = qjkZ$ levels, respectively, where $\sum N_k = N$ and $\sum Z_k = Z$. Only in this case where N_k particles are brought into the separate parts Z_k of the Z-space, does the redistribution have the same statistical weight as the original distribution. (2) From the same original distribution is obtained a redistribution in which $N_k = q_{jk}N$ particles are now allowed in the whole Z-space of all Z levels. This redistribution belongs to an increased statistical weight. It is obtained from case 1 by a diffusion of each group of N_k particles from the partial volume Z_k to the total volume Z. One may call case 1 "coherent" and case 2 "incoherent redistribution." The distinction is of importance for problems of reaching thermal equilibrium. When two dilute gases A and B diffuse from separate volumes V into the common volume $2V$, case 2 prevails, and the maximum isothermal work is

$$W = nRT\{2\ln 2 - (1+q)\ln(1+q) - (1-q)\ln(1-q)\} \quad (3)$$

with value 0 for $q=1$, and value $2nRT\cdot\ln 2$ for $q=0$. Equation (3) determines the function $f(q)$ left open in Eq. (1″).[1]

7. SUPERPOSITION OF PROBABILITY AMPLITUDES

Suppose we construct a table or matrix, usually rectangular, of the fractions $q(A_k, B_j)$:

$$\begin{Bmatrix} q(A_1,B_1) & q(A_1,B_2) & q(A_1,B_3) & \cdots \\ q(A_2,B_1) & q(A_2,B_2) & q(A_2,B_3) & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{Bmatrix}$$

and similar matrices for $q(B, C)$ and $q(A, C)$, and so forth. The question arises whether there are quantitative relations between the elements of the various matrices, in particular whether $q(A, C)$ depends on $q(A, B)$ and $q(B, C)$.

Let us first consider an analogous problem. Suppose a number of objects called $A, B, C, \cdots$ are linked by quantities q_{AB}, q_{BC}, q_{AC}, and so forth, all q's having positive real values between 0 and 1, including the special values $q_{AA}=0$, etc., for "self-linkage." We now want to establish a mathematical relation between the q's so that a given q_{AB} and q_{BC} will determine q_{AC}, possibly in a multivalent fashion. The mathematical relation is required to be symmetric and self-reproducing so that it will also yield q_{BC} when q_{AB} and q_{AC} are given; furthermore, in the case of A being identical with C, the relation $q_{AA}=0$ shall result as a special case. In spite of the vagueness of the problem, the restrictions imposed on the interrelations are pointing to a definite solution. Indeed, the symmetry require-

[1] For a more detailed discussion refer to A. Landé, Phys. Rev. (to be published).

ment together with the q's all being less than unity offers the interpretation of the q's as distances between points A, B, C, $\cdots$ located on a circle of diameter unity; hence q_{AB} and q_{BC} will determine q_{AC}, although in a bivalent fashion, since C may be on either side of B. More generally, q may be taken as the geodesic distance between points on a circle according to any metric of axial symmetry, e.g., over a dome topping the circle. It is hard to imagine any other possible solution of the interrelation problem under the above restrictions; however, it would be interesting to find a strict proof of uniqueness.

We now turn to the quantum problem, again concerning objects A, B, C, $\cdots$ linked by quantities q_{AB}, etc. However, the objects now have parts, A consists of members A_1, A_2, $\cdots$ also connected by q's, of value $q_{A_kA_{k'}}=\delta_{kk'}$; i.e., each object is self-orthogonal. As to the $q_{A_kB_j}$, they all are positive between 0 and 1, and satisfy the unit-sum rule (2′) of rows and columns: $\sum_k q_{A_kB_j}=1$ as well as $\sum_j q_{A_kB_j}=1$. Wanted is a mathematical relation between the q's so that given values of the q_{AB} and q_{BC} will determine the values of the q_{AC} in a symmetric and self-reproducing, although possibly multivalent fashion. The relation also is required to result in $q_{A_kA_{k'}}=\delta_{kk'}$ in the special case of A being identical with C. Again, in spite of the vagueness of the problem, the imposed restrictions lead to a definite solution. The unit-sum rule (2′) for the q-matrices definitely points to the interpretation of the q's as cosine-squares between "axis directions" A_k and B_j of orthogonal coordinate systems A and B, suggesting the interrelation $(q_{A_kC_m})^{\frac{1}{2}}=\sum_j (q_{A_kB_j})^{\frac{1}{2}}\cdot(q_{B_jC_m})^{\frac{1}{2}}$ in which the square roots have undetermined signs. The choice of $\pm$ signs is restricted by the requirement that the product of one row with another, or one column with another, is to yield the sum *zero* in every $\sqrt{q}$-matrix. One also may introduce the generalization that the q's are the absolute squares of "Hermitian cosines" ψ as intermediate quantities.

$$\psi_{A_kB_j}=+(q_{A_kB_j})^{\frac{1}{2}}\exp(i\varphi_{A_kB_j})$$

linked by the relation

$$\psi_{A_kC_m}=\sum_j \psi_{A_kB_j}\psi_{B_jC_m}. \tag{4}$$

The multivalence of the phases then is restricted by the requirement that in every ψ-matrix the sum of the products of the elements of one row with the complex conjugates of another is *zero*; the same rule holds for the columns. Just as in the former example, it is hard to see how one could construct another solution of the interrelation problem between the q's with the restrictions imposed on them. One may therefore say tentatively that the introduction of complex amplitudes ψ and their combination law, i.e., the Born-Heisenberg-Jordan theory, is a direct mathematical consequence of fractional likeness between states and thereby a consequence of thermodynamic continuity. Still, a strict proof of the uniqueness of this method of solving the interrelation problem would be welcome.

8. QUANTUM MECHANICS, DUALITY, UNCERTAINTY

The results developed so far constitute *general* quantum theory. *Special* quantum mechanics asks the more specific question: What is the magnitude of the likeness factors q_{kj} between states A_k and B_j of a mechanical system in which the states A and B refer to certain functions $A(q, p)$ and $B(q, p)$ of coordinates q and momenta p? Answering this question requires one more bit of information, namely, knowledge of the amplitude ψ of likeness between a state of given coordinate value q, and a state of given value p of the associate momentum. This amplitude has been found inductively to be

$$\psi(q, p)=\exp(2i\pi qp/h), \tag{5}$$

a function symmetric and periodic in q and p introducing Planck's quantum h into the theory. This periodic function is responsible for the famous duality of particles and waves: As a function of q the amplitude ψ is periodic in space with wavelength $\lambda=h/p$. Furthermore, according to wave theory, the number of waves per unit length, $1/\lambda$, can be determined with accuracy $\Delta(1/\lambda)\sim 1/\Delta q$, if Δq is the range over which the wave train is allowed to be tested. Replacing $1/\lambda$ by its original meaning p/h again, the last relation reads $\Delta(p/h)\sim 1/\Delta q$, or $\Delta p\cdot\Delta q\sim h$, i.e., the uncertainty relation of Heisenberg for the margins of accuracy of a simultaneous determination of the values q and p.

Apart from the special quantum-mechanical results based on the knowledge of the function $\psi(q, p)$ of Eq. (5), all the general conceptions and theorems of quantum theory, those concerning discrete sets of mutually orthogonal states with fractional likeness between states in general, the probability interpretation of the likeness fractions q, the splitting effect with conservation of statistical weight, i.e., quantum statistics, and last but not least, the mathematical, i.e., logical, necessity of introducing probability amplitudes ψ linked together by the matrix multiplication law (4)—all these results are immediate consequences of the postulate of thermodynamic continuity: "the Gibbs paradox does not occur." One may speculate on what would have been the development of physics if Gibbs in the 1890's had consistently pursued his objection to the discontinuity inherent in the classical theory of diffusion.

An Absolute Ampere Current Balance for Laboratory Use

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(Received February 12, 1952)

The ampere current balance herein described has been in use for four years as a regular experiment of a sophomore physics course. It has the following merits: (1) It is simple to construct; (2) it is absolute in that the current may be calculated from the dimensions and balancing weight only; (3) it is subject to a high degree of accuracy, especially for the larger currents; (4) it may be used for alternating as well as direct currents; (5) when current is passed through the movable conductor only, the balance may be used to measure the horizontal component of the earth's magnetic field.

1. INTRODUCTION

THE ampere is now defined in terms of the force between conductors.[1] This being so, it becomes necessary to construct balances having current-carrying conductors whose dimensions are such that accurate calculations may be made so that the values of the currents may be computed in terms of these dimensions and the balancing weights. Such balances exist, for example, at the Bureau of Standards in Washington.[2]

Elaborate pieces of equipment such as these are of course not suited for ordinary laboratory use, but none the less it is highly desirable to be able to make such measurements because of the importance of the principles involved. Ampere balances suitable for measuring *relative* currents are in common use. Described below is an *absolute* balance of simple construction that is used in a sophomore physics laboratory.

[1] F. B. Silsbee, "Establishment and Maintenance of the Electrical Units," Natl. Bur. Standards Circular 475 (1949).
[2] Curtis, Driscoll, and Critchfield, J. Research Natl. Bur. Standards **28**, 133 (1942), RP 1449.

2. GENERAL DESCRIPTION

For students of the sophomore level to comprehend properly, it is desirable to be able to use the simple equation derived from the law of Biot and Savart, $F/l=\mu_0 i_1 i_2/2\pi a$, (for the force F between parallel conductors of lengths l, carrying currents i_1, i_2 when separated by a distance a in free space of permeability μ_0), as the basic equation in calculating the current. In order that this should strictly apply, the radius of curvature, for example, of any conductors must be large compared with the distances between conductors. The simplest geometrical shape which at the same time can be made sensitive to small torques, is a uniform circular conductor which is able to rotate about a diameter. Another concentric conductor with the same radius of curvature, whose plane is parallel to but displaced a small distance from this one, will produce a net torque provided current flows parallel in the two adjacent conductors around one-half the loop and antiparallel around the other half.

Aside from the difficulties of making currents flow like this, a further practical disadvantage

comes in not being able to measure conveniently the distance between centers of the conductors. These two difficulties are solved by placing at an equal distance on the other side of the movable conductor another concentric split ring as shown in Fig. 1, and so connected that the currents flow in the proper directions. The torques now add up between the stationary conductors and all parts of the pivoted, central conductor and the same current, in magnitude, flows in each.

The important spacing now becomes that between the two outer conductors. Consider the three parallel conductors with currents flowing, as shown in Fig. 2, giving a force on the central conductor as indicated. Let the center conductor be displaced an amount Δa from its central position. As the movable conductor comes closer to the upper one, the force will increase, but there will be a decrease in force due to the current in the lower conductor. The force per unit length on the center one will be

$$\frac{F}{l}=\frac{\mu_0 i^2}{2\pi}\left[\frac{1}{a+\Delta a}+\frac{1}{a-\Delta a}\right]$$

$$=\frac{\mu_0 i^2}{2\pi a}\left[1-\frac{\Delta a}{a}+\left(\frac{\Delta a}{a}\right)^2 -\cdots+1+\frac{\Delta a}{a}+\left(\frac{\Delta a}{a}\right)^2+\cdots\right];$$

and if $\Delta a \ll a$,

$$\frac{F}{l}=\frac{\mu_0 i^2}{\pi a}\left[1+\left(\frac{\Delta a}{a}\right)^2\right] \text{ to second-order terms.} \quad (1)$$

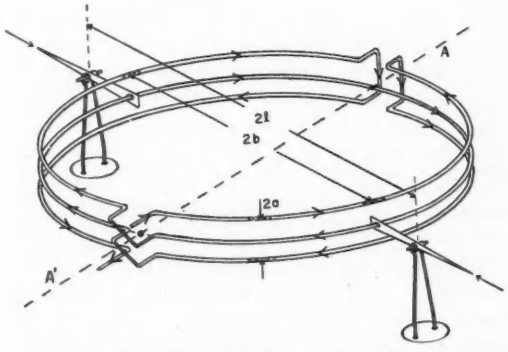

FIG. 1. Schematic drawing of current balance showing one solution of connections for currents to give torques on movable, central conductor, in the same direction.

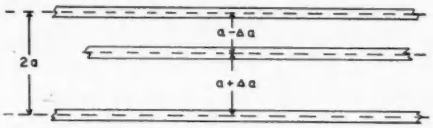

FIG. 2. The force on the middle movable conductor is to be calculated. The movable conductor lies between two fixed conductors a distance $2a$ apart. Equation (1) shows that the position of the middle conductor is unimportant unless Δa is greater than a few percent of a.

Thus, the position of the center conductor is unimportant until Δa becomes of the order of 5 or 10 percent of a. The important spacing is the distance between centers of the fixed, outer conductors and this distance can be measured accurately with micrometer calipers.

3. DETAILED DESCRIPTION

a. The Fixed and Movable Conductors

In the instrument as constructed, the diameter of all the circular conductors is 50.0 cm, and they are made of $\frac{3}{16}$-inch copper tubing. The distance between outer conductors is approximately 2.7 cm, or just slightly too great for the usual one-inch micrometer caliper to fit over. The distance between the outer conductors is accurately determined by eight Lucite spacers, as shown in Figs. 3 and 4. These are all machined while clamped together. The micrometer calipers do fit across the parallel faces on the Lucite spacers, and these are much less easily distorted by the calipers than are the circular conductors. The distance between centers of the outer conductors is then determined by adding to this dimension the radius of each conductor. Connections to the four semicircular stationary loops are brought out to large binding posts mounted on two Bakelite panels. These terminals, including those from the movable conductor, are equally spaced so that slotted strap conductors may be used to provide the proper connections as shown in Fig. 3. An important part of the experiment for the student is to work out the proper arrangement of these straps at both terminal panels so that the torques are all in the same direction on the movable conductor.

The movable conductor is made in two halves with a portion of a single-edge razor blade serving as the knife-edge at each end of a diameter where

the two halves join. The details of that junction, where the current is brought in and goes out, are shown in Fig. 4. The knife-edge itself is just slightly above the center of the conductors forming the central loop. The knife-edge, necessary mica insulation, and conductors that lead to shallow mercury cups, are all clamped together by the four small bolts seen in the middle of Fig. 4.

The knife-edges rest in V-shaped grooves. That one at the junction, where the current enters and leaves, is midway between the two uprights that contain the mercury contacts. The line joining the knife-edges must be slightly above the center of mass of the movable system if the system is to be in stable equilibrium. It has been found that when the period of oscillation is about five seconds, the disturbances caused by the usual air currents in the laboratory are not noticeable.

The conductors that lead to and from the ends of the movable conductor to the mercury cups are made of copper and are heavily nickel plated. Their lower ends that dip into the mercury are in the same straight line as the knife-edges. This alignment tends to minimize any torques due to the mercury surface.

The mercury is contained in shallow cups made by bending thin nickel sheet over a rectangular form, thus making folded corners. The cups are soldered into the ends of brass uprights. The conductors that dip into the mercury reach within about one millimeter of the bottom of the cups, thus keeping the path of the current through the mercury to a minimum. All metal parts are heavily nickel plated before final assembly.

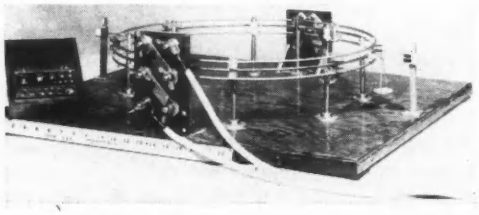

FIG. 3. Slotted, strap connectors are provided for each terminal board. The student is expected to find a combination of connections that will give a resultant torque on the central conductor, due to each half-loop, in the same direction.

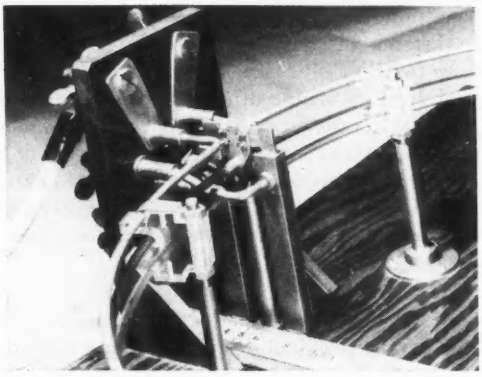

FIG. 4. Enlarged view of the rear of the main terminal board.

The above construction has proven very satisfactory. No frictional effects in the mercury contacts have been noticed; also very little heating occurs even when currents of 50 amperes are carried for some time. Further, no apparent amalgamation of the mercury with other metal parts has taken place in spite of the fact that the mercury has been in the cups continuously for four years.

b. Weight Pans and Pointers

To provide a proper torque to balance the torque due to the current, two identical weight pans and pointers with mirrors and scales are provided as shown in Figs. 1 and 3. The weight pans are supported by hardened steel triangular knife-edges soldered into the pointers.

The dual pointers allow two observers to watch simultaneously. The two weight pans are necessary to find the average lever arm. Thus, in practice, a mass is placed in one pan, and the current is increased until a balance is obtained. Then the current in the movable loop is reversed, the mass is changed to the other pan and a new balancing current is read. Then if (l_1+l_2) is the distance between knife-edges, $\frac{1}{2}(l_1+l_2)=l$ will be the average lever arm; and if $l_1 \approx l_2$, the true current i can be shown to be the average of the currents i_1 and i_2. For the two pieces of apparatus constructed, the two lever arms are so nearly the same that the two currents are the same to within less than one part in 500. (See examples below.)

c. Auxiliary Apparatus

The experiment, as performed, regards the ampere balance as a standard and an ammeter is calibrated in terms of the values calculated from measurements with the balance. A 0–50 ammeter is quite satisfactory. In addition, a resistor, such as a carbon pile that has a low resistance and gives a smooth, continuous adjustment, is desirable. As a source of current, a 2-volt lead storage cell is quite sufficient. For weights, those used with the usual analytical balance are suitable. It is also necessary to provide connectors that will carry the necessary currents without having too large iR drops.

4. CALCULATIONS

To calculate the torque on the movable conductor, use is made of the law of Biot and Savart, combined with the expression for the force per unit length on a conductor carrying a current in a magnetic field. Using the rationalized mks system of units, the torque on the movable conductor about the diameter AA', owing to the stationary conductors when all the currents are numerically the same and the torques are all in the same direction, is easily shown to be

$$L=4\mu_0 i^2 b^2/\pi a, \tag{2}$$

where μ_0 has the value $4\pi\times10^{-7}$ webers per ampere-meter, i is the common current, b the common radius of all loops, and $2a$ the distance between the centers of the outer loops.

When this torque is balanced by the torque due to the added weights mgl, the current will then be given by

$$i=\left(\frac{\pi}{4}\frac{mgla}{\mu_0 b^2}\right)^{\frac{1}{2}}=km^{\frac{1}{2}}. \tag{3}$$

The above calculation using the law of Biot and Savart is based on the assumption that the conductors are straight and very long compared with the distances under consideration. For a more accurate calculation, one may refer to Smythe's book.[3] For two concentric circular loops of the same radius b, distance between planes a, each carrying a current i, the mutual force is

$$F=\frac{\mu_0 i^2 a}{(4b^2+a^2)^{\frac{1}{2}}}\left[-K+\frac{(2b^2+a^2)E}{a^2}\right],$$

where K and E are complete elliptic integrals of the first and second kind, respectively, with modulus $k^2=4b^2/(4b^2+a^2)$. Using the values $b=50$ cm and $a=1.45$ cm, the force thus calculated is one-half percent less than that calculated by using the simple law of Biot and Savart.

It should be emphasized that both calculations neglect the fact that in the actual apparatus the loops are not complete, and that there are other conductors going to the terminal boards. There are apparently compensating effects, for the actual current compared with that calculated from the simple formula is well within the error that might be expected due to the curvature of the conductors as will be seen in the examples cited below.

5. PROCEDURES AND EXAMPLES

The student is given the balance without the connecting straps on the terminal boards being in place. From his knowledge of the direction of forces on conductors in magnetic fields he is then expected to arrive at solutions which give (1) the same absolute value of current in all conductors, (2) all the torques on the movable conductor in the same direction, and (3) the reverse direction of torque on the movable conductor. This last is necessary to find the average length of lever arm. One solution is that shown in Fig. 2. The torque can be reversed by changing the positions of the connecting straps on only one terminal panel.

In use, the axis of rotation of the movable loop is pointed magnetic north and south to avoid the torque due to the earth's magnetic field. The diameter of the loops is measured at several places by placing a meter stick on its edge on the top stationary loop and noting the points of contact with the $\frac{3}{16}$-inch conductors. This distance can thus be measured to ±0.2 mm. The distance between knife-edges that hold the weight pans is also determined when the meter stick is on edge across the top stationary loop. The usual meter stick has mm markings along the two sides, thus allowing the observer to sight along similar markings to the knife edges of the weight pans. This distance may thus be measured to ±0.2 mm.

[3] Smythe, *Static and Dynamic Electricity* (McGraw-Hill Book Company, Inc., New York, 1939), p. 277.

Examples as a Current Measuring Instrument

The current balance was compared against a Weston Model 5, standard ammeter with a 12.5-inch scale. This instrument had been recently calibrated with a potentiometer and standard cell. Constants of the ampere balance:

twice the lever arm $= 2l = 0.5559 \pm 0.0002$m,
twice the radius of the loops $= 2b = 0.4994 \pm 0.0002$m,
distance between centers of outer conductors $= 2a = 0.02898 \pm 0.00002$m.

The constant k, given by Eq. (3), is then

$$k = \left(\frac{\pi a g l}{4\mu_0 b^2}\right)^{\frac{1}{2}} = 628.8 \pm 0.4 \text{ amp kg}^{-\frac{1}{2}},$$

if m is in kg, or

$$k = 19.885 \pm 0.012 \text{ amp g}^{-\frac{1}{2}},$$

if m is in grams.

	Trial No. 1	Trial No. 2
Mass in pans (g)	4.000	9.000
Reading of st'd ammeter (amp)	39.725	59.595
Reading of st'd ammeter with current reversed (amp)	39.725	59.695
Average current (amp)	39.725	59.645
Current computed from constant of current balance, $I = km^{\frac{1}{2}}$ (amp)	39.770	59.655
Percent difference	0.14	0.016

At currents from 40 to 60 amperes, the period of oscillation noticeably increases. At the larger value, instability has nearly set in, resulting in a large displacement of the movable conductor for changes of current that are not noticeable on the ammeter.

The current balance is accurate to about ± 1 percent for currents of 8 to 10 amperes, while below 2 amperes the accuracy is not better than ± 5 percent due to the small torques involved. (A balancing weight of 10 mg corresponds to 2 amperes.) For currents from 20 to 60 amperes the balance is accurate to ± 0.2 percent.

The instrument may also be used to measure the horizontal component of the earth's magnetic field. When so used, the axis of rotation of the movable conductor is pointed magnetic east and west and current is sent through it only. Stray magnetic fields must now be guarded against. The following procedures have been found sufficient to eliminate the effects of stray fields due to currents flowing in connecting conductors: (1) use either a concentric line or a twisted pair to the movable conductor; (2) reverse the current and take the average value. With these precautions an extended magnetic field of 0.25 gauss may be measured to ± 2 percent.

SUMMARY

The current balance described above has proven to be most satisfactory as a precision instrument for measuring currents between 15 and 60 amperes. After four years' use by the sophomore class, the two instruments have required no adjustments, and their present constants are very close to their original values. No precision shopwork is required on any of the parts in their fabrication, except on the Lucite spacers. These should be made with some care to insure uniformity and to provide parallel faces for micrometer measurements.

Pedagogically the instrument has been found useful in helping the student (1) apply his knowledge of forces on current-carrying conductors in magnetic fields, (2) gain a feeling for the magnitude of the forces thus involved, (3) test his ability to make accurate measurements.

In conclusion the author wishes to thank Mr. T. Harvey for taking the photographs which are reproduced in Figs. 3 and 4.

Among the admirable American expressions which enter our language from time to time to lend it new vitality, there is one which I would have every audience, and every lecturer, use at the end of every lecture or course: So what? *What does all this amount to?*—T. R. HENN, The Apple and The Spectroscope (Methuen, 1951) by permission.

Path Concepts in Hamilton-Jacobi Theory

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(Received November 5, 1951)

It is pointed out that the classical analog to a wave function is not a single orbit but a set of paths as defined by the solution of the Hamilton-Jacobi equation. A review is given of Hamilton-Jacobi theory in language appropriate to these path concepts. It is shown that there is a classical analog to a wave packet. Quantum-mechanically, constants of motion are not arbitrary but must be characteristic values of commuting operators. It is shown by examples that the possible classical states are similarly restricted. The conditions are discussed under which a classical set of paths is derivable from the solution of a Hamilton-Jacobi equation.

AS a result of the rather formal treatment usually accorded to Hamilton-Jacobi theory and the theory of contact transformations, some of the important features of this theory are frequently left unappreciated. Much of this lack of appreciation arises from the association of the theory with the older quantum mechanics. Sommerfeld-Wilson quantization was concerned with selecting one particular orbit. Consider, for example, the problem of the hydrogen atom. Here the older quantum theory characterized the orbit by the three quantum numbers n_r, m, and l. These are not adequate to really specify a particular orbit. If one particular orbit is taken and then permitted to precess either about the z axis or about the direction of the total angular momentum vector, other equally acceptable orbits are obtained. It was usually implicitly assumed that the electron occupied only one of these orbits. In wave mechanics we realize that the uncertainty principle must be satisfied. Motion along a well-defined orbit is then out of the question. To get a reasonable approximation[1] to the behavior of the electron as predicted by the wave function, it becomes necessary to assume that all of the classical orbits which correspond to the same n_r, l, and m, have an equal probability of being occupied.

This view is really much more in accord with the picture suggested by Hamilton-Jacobi theory. The solution S of the Hamilton-Jacobi equation is always associated with a whole assembly of paths rather than a single orbit. If this viewpoint is kept clearly in mind and the wave function always compared to a set of paths rather than a single one, many of the presumed differences between quantum mechanics and classical mechanics disappear. The new formulation of quantum mechanics given by Feynman[2] makes this correspondence particularly easy to understand.

It is particularly important that the restrictions governing the solution of the Hamilton-Jacobi equation be understood. This paper is an attempt to illustrate these restrictions. The existence of such restrictions can be realized from the fact that $A\exp(iS/\hbar)$ is an approximate solution of the (possibly time dependent) Schroedinger equation. Here S is the solution of the corresponding classical Hamilton-Jacobi equation; A^2 is the density of a swarm of classical particles moving along the paths corresponding to S. If the density distribution A^2 is given at some initial time, then thereafter A^2 will be determined if S and therefore the paths are known. Hence, a knowledge of S determines the approximation to the wave function. It is only reasonable therefore that properties of the wave function be reflected in corresponding properties of S.

It is the purpose of the present paper to give a more pictorial discussion than is usually presented of the relationship between the Hamilton-Jacobi equation and the wave equation. This is done in accordance with the commonly accepted

* Now with the International Business Machines Corporation.

[1] A detailed discussion of such approximations is given in the author's thesis, "Phase Integral Approximations in Wave Mechanics," submitted to Harvard University, 1950. See also J. H. Van Vleck, Proc. Natl. Acad. Sci. **14**, 178 (1928).

[2] R. P. Feynman, Revs. Modern Phys. **20**, 367 (1948).

concepts, and there is no attempt made to reinterpret the statistical nature of wave mechanics. Such a reinterpretation was recently given by D. Bohm.[3] The paths mentioned in the present paper are determined in a strictly classical fashion and are not affected by Bohm's "quantum-mechanical potential." The function $A\exp(iS/\hbar)$ used in the present paper is an approximation to the wave function, in contrast to Bohm's $R\exp(iS/\hbar)$ which is exactly equal to the wave function. Furthermore, if several waves are superposed, as in the case of a standing wave, we assume that each separate component wave is approximated by a function of the form $A\exp(iS/\hbar)$, whereas Bohm sets the whole wave function equal to $R\exp(iS/\hbar)$. The S used here will be complex, outside of the region which is accessible to the classical particle; Bohm's S is always real.

REVIEW OF USUAL THEORY

The Hamilton-Jacobi equation is

$$(\partial S/\partial t) = -(\nabla S)^2/2m - V, \tag{1}$$

where V is the potential in which the swarm of particles moves. The word "swarm" is used very purposefully. In addition, S is a function of time and of the coordinates x, y, and z. A real solution of S may exist over only part of space, but it always does exist over a region, not just at a single point. A single moving particle exists only at one point. It is clear therefore that S must be associated with many paths, not just one. The particles which move along these paths (which may be imagined infinitesimally separated from each other) constitute our "swarm" [or "assembly" if a more conventional word is desired]. Each of the particles is moving in the same potential. The momentum of the particles is given as a function of position and time by

$$\mathbf{p} = \nabla S. \tag{2}$$

Equation (1) tells us how S changes with time at a fixed point in space. If instead of staying at a fixed point we move along with a particle, then the rate of change of S with time is given by

$$dS/dt = L = T - V = (\nabla S)^2/2m - V. \tag{3}$$

By using either Eq. (1) or Eq. (3) we can follow the change of S with time. At each instant ∇S will give us the momentum distribution in space. Consider the momentum vector at a certain point represented by an arrow at that point. It shows the direction in which the particle is moving at that time. At a little later time the particle will have moved in that direction. The value of S meanwhile will have changed a little, as indicated by Eq. (1). Since the value of S has changed and since the particle also has moved to a new point, the value of ∇S at its new point will be a little different from what it was originally. The new ∇S again gives us the direction in which the particle will move. By thus joining the arrows, which at successive instants represent the momentum of a particle, we obtain the path of that particle. This graphical method is equivalent to the solution of a differential equation. We see then that if S has been found by solving Eqs. (1) or (3), a second differential equation must be solved to find a path. If we are given S as a function of the coordinates at an initial time and again at a later time, this in itself does not give us the solution of a mechanical problem. It does tell us how the particles of the swarm were moving at the initial time and again at the final time. It does not, however, tell us which part of the swarm at the final instant corresponds to which part of the swarm at the initial instant. It does not tell us to what point a particular particle moved.

To do this, another procedure is usually used. Let the function S at the initial time be $S(x, y, z, t_0)$. This together with Eq. (1) defines S at all times t. Now consider what happens if the function S at t_0 is changed slightly to $S' = S(x, y, z, t_0) + \delta(x, y, z, t_0)$. Then the solution at later times will also change. Let the change be denoted by δ so that

$$S' = S(x, y, z, t) + \delta(x, y, z, t). \tag{4}$$

It is then shown in the usual theory that $\delta(x, y, z, t)$ has a constant value along any mechanical path defined by the original function S. The particles, which at t_0 were at the particular points where $\delta(x, y, z, t_0)$ had the value δ_0, will at a later time t_1 be at the points where $\delta(x, y, z, t_1)$ has the value δ_0. Consider the particular three-dimensional situation which our notation has implied. In that case the set of points for which $\delta(x, y, z, t_0) = \delta_0$ will form a two-dimensional surface. Therefore, comparing the solution S with

[3] D. Bohm, Phys. Rev. **85**, 166 (1952).

the solution S' will only tell us that the particles that were on a certain surface at t_0 have moved to a different surface at t_1. It still does not enable us to tell the initial and final points for each particle. Let us now, however, in addition to S', consider two other solutions

$$\begin{aligned} S'' &= S(x, y, z, t) + \epsilon(x, y, z, t) \\ S''' &= S(x, y, z, t) + \gamma(x, y, z, t), \end{aligned} \tag{5}$$

where ϵ and γ just like δ are small changes. Now each point at t_0 has three values δ, ϵ, and γ associated with it. Therefore δ, ϵ, and γ form a coordinate system which enables us to assign a particular set of coordinates to each point. (As in the selection of any coordinate system, care must be taken so that no two points are assigned the same coordinates.) Also δ, ϵ, and γ have constant values along the mechanical paths defined by S. A knowledge of δ, ϵ, and γ therefore gives us the explicit analytical solution for the motion of all the particles in the swarm.

It is to be particularly noted that δ, ϵ, and γ are arbitrary functions; they are not determined by S or by the Hamiltonian. They should of course be small; or even more exactly, where we have talked about changes, one should really talk about derivatives.

The statements made in this section are only a review of the usual theory. While the language in which the theory is stated may not be quite the usual, the ordinary proofs apply.[4] If, however, we are satisfied with considering the motion in a Cartesian space of a finite number of dimensions, then the proofs are all quite trivial and no consideration of contact transformations is necessary.

COMPARISON WITH WAVE MECHANICS

We have up to now stated what happens when we are given the value of S at an initial time. This corresponds to the situation in wave mechanics where the initial value of a time-dependent wave function is given. In wave mechanics this is not the only method by which a state is defined. Instead, we can specify the wave function at a certain time by its characteristic values with respect to some complete set of commuting operators. Let $\mathbf{A}$ be one of these operators. These operators are then constants of motion, not necessarily in time, but in the sense that at the particular instant at which the wave function is a characteristic function of the operators, we have

$$\int \psi^* \mathbf{A}^2 \psi - \left(\int \psi^* \mathbf{A} \psi \right)^2 = 0. \tag{6}$$

(If the Hamiltonian is not one of the commuting set, ψ will not remain a characteristic function of $\mathbf{A}$. Equation (6) then applies only at the instant at which ψ is a characteristic function of the specified operators.) A measurement of $\mathbf{A}$ on the specified state at the specified time will always yield the same result, namely, the characteristic value. In setting up a classical analogy to our state, we must require that all particles in the swarm have the same value of whatever classical quantity corresponds to the operator $\mathbf{A}$.

Most frequently the Hamiltonian $\mathbf{H}$ is chosen as one of the complete set of operators. In the classical analog we then expect all of the particles to have the same energy. Hence,

$$\partial S/\partial t = -(\nabla S)^2/2m - V = -E \tag{7}$$

is independent of the space coordinates, and we have the time-independent Hamilton-Jacobi equation in correspondence to the time-independent Schroedinger equation. The solution of Eq. (7) is

$$S = S(x, y, z, t_0) - E(t - t_0). \tag{8}$$

Therefore ∇S is time independent, and the momentum distribution in space does not change in time. Each particle in the swarm follows along the tracks of an earlier particle.

In quantum mechanics if we wish to follow the motion of a localized particle (or at least as well-localized as the uncertainty principle will permit us), the wave function for a stationary state does not tell us much. We have to form a wave packet involving a narrow range of solutions. We have seen that the classical situation is very similar. A solution of the form of Eq. (8) will not tell us directly how a localized particle moves; we must instead consider a range of solutions, corresponding to δ, ϵ, and γ of the previous section. The classical calculation can be made to resemble the wave mechanical one even more strongly by not using three functions, δ, ϵ, γ, but only one

[4] See for example, H. Goldstein, *Classical Mechanics* (Addison-Wesley Press, Inc., Cambridge, Massachusetts, 1950.)

function. Let this be β. Choose β to be vanishing except in a small neighborhood, so that

$$S' = S + \beta \tag{9}$$

is different from S only in a small neighborhood. As time goes on, the region in which S' differs from S will move, showing how the particles in that particular region move.

In wave mechanics we cannot arbitrarily choose the operators which specify a state; the operators must commute. What happens in the classical case? Consider a two-dimensional example. The operators $(\hbar/i)(\partial/\partial\theta)$ and $(\hbar/i)(\partial/\partial y)$ do not commute. They therefore do not define a complete set of states. Indeed, except for the case of a constant, they do not have a single characteristic function in common.

The classical quantities corresponding to $(\hbar/i)(\partial/\partial\theta)$ and $(\hbar/i)(\partial/\partial y)$ are the angular momentum M_z and the momentum component p_y, respectively. If the values of p_y and M_z are held fixed, then p_x can be found from

$$xp_y - yp_x = M_z \tag{10}$$

or

$$p_x = (xp_y - M_z)/y. \tag{11}$$

There is no trouble therefore in assigning a momentum (p_x, p_y) to every point, such that the prescribed values of M_z and p_y hold throughout the plane.

It is, however, not just a momentum distribution that is analogous to a wave function but a Hamilton-Jacobi S function. Does the momentum distribution we have set up correspond to an S function so that $\mathbf{p} = \nabla S$? In order for this to b possible, we must have

$$(\partial p_y/\partial x) = (\partial p_x/\partial y). \tag{12}$$

Our momentum distribution gives

$$(\partial p_y/\partial x) = 0, \tag{13}$$

$$(\partial p_x/\partial y) = -(xp_y - M_z)/y^2, \tag{14}$$

which clearly does not satisfy Eq. (12) (except for the trivial case $p_y = M_z = 0$, which corresponds to a constant wave function).

We see then that Eq. (12) is a restrictive condition which prevents us from choosing our constants of motion arbitrarily. While it is possible to set up momentum distributions which have no quantum-mechanical equivalent, it is presumably not possible to set up Hamilton-Jacobi functions in these cases.

This statement involves the realization that there are momentum distributions which can never, even for only an instant of time, be reached by a swarm of particles obeying a Hamilton-Jacobi equation.

Consider, for example, the case of motion in a central force field. Quantum-mechanically $\mathbf{M}^2$, $\mathbf{M}_z$, and $\mathbf{H}$ are usually chosen as constants of the motion. On the other hand, $\mathbf{H}$, $\mathbf{M}_z$, and $\mathbf{M}_y$, for example, cannot be chosen since $\mathbf{M}_z$ and $\mathbf{M}_y$ do not commute. What happens in the classical case? If M^2, M_z, and H are given prescribed values, then a momentum distribution can easily be found. The solution for the momentum satisfies

$$\nabla \times \mathbf{p} = 0, \tag{15}$$

so that $\mathbf{p}$ is the gradient of an S function. If H, M_z, and M_y are instead chosen as constants of the motion, a set of paths with the correct momentum distribution again exists, but it does not satisfy Eq. (15).

EXISTENCE THEOREMS

We have seen that a classical momentum distribution does not in general have an S function associated with it. This section will examine some simple situations in greater detail to show when an S function exists. In general, the necessary and sufficient criterion is

$$(\partial p_i/\partial x_j) = (\partial p_j/\partial x_i). \tag{16}$$

In a one-dimensional case an S function always exists, since for any momentum distribution p_x, there is always an $S(x)$ such that

$$\partial S/\partial x = p_x.$$

If a system of particles moving in a fixed potential can be described by an S function at one particular instant then by solving Eq. (1) the S function that describes the system at any later time can be found.

In two dimensions we have already shown by an example that not every momentum distribution has an S function. If, however, we specify a set of paths, all corresponding to motion with the same energy, in the same potential and use these to define our momentum distribution, then an S

function will exist. To show this, consider an arbitrary line crossing the selected set of paths. Along this line the variation of S is defined by the component of $\mathbf{p}$ lying along the line. The variation of S perpendicular to the line can be found from

$$(\partial S/\partial a)^2+(\partial S/\partial n)^2=2m(E-V), \qquad (17)$$

where $\partial S/\partial a$ and $\partial S/\partial n$ are, respectively, the derivatives of S moving along and normal to the line. By moving a small distance along the normal to our arbitrary line and using the value of $\partial S/\partial n$ from Eq. (17) we can find S on a new line. Since we know S on the new line, we know $\partial S/\partial a$ on it and therefore by Eq. (17), $\partial S/\partial n$ again also. We can therefore extrapolate to still another line and continuing in this fashion cover the whole region. This gives us an S function and one which certainly satisfies Eq. (17). Is it one appropriate to our original paths? The paths defined by the S function obtained by our extrapolation method and those originally specified are both solutions for the same potential. Each path obtained by the extrapolation method agrees in position and momentum with one of the originally specified paths at one point. This point is on the arbitrarily chosen line that crosses the specified paths. But giving the momentum at one position defines the path uniquely. Hence, the two systems of paths are identical, and the S function we constructed is appropriate to the paths originally specified.

In two dimensions we have just seen that if a set of paths all correspond to the same energy and potential, an S function must exist. This is not true any more in three dimensions. One example that has already been given is the case of the central force problem in which E, M_z, and M_y are prescribed. A much simpler example can, however, be given. Consider a set of planes, $z=$constant. In each plane let there be a set of particles filling the plane and all streaming in the same direction. Let this direction, however, vary from plane to plane. Since the momentum has no z component anywhere $\partial S/\partial z=0$. Clearly, however, S must vary from plane to plane. Hence, no S function can exist.

Quite apart from the broadening of outlook, there is another reason why the study of literature is of essential importance to the scientist. Some scientists may have been content to seek knowledge for their own mental satisfaction alone, with complete indifference to imparting their experience to others; they correspond to the ascetic. But they are rare, and the normal scientist finds the deepest pleasure in exchanging his thoughts with his fellow men. Science is becoming so vast and specialized that it is increasingly difficult to keep abreast of development. Just as gold is mined from the earth with infinite labour to be buried again in the earth in vaults, so new knowledge may only too easily be buried in scientific journals until with the lapse of time it can no longer be of interest or influence the progress of science. It is as important to present new knowledge in a form in which it can be assimilated and its essential import realized as it is to discover it, and this presentation is an art akin to poetry and literature.—W. L. BRAGG, Foreword to The Apple and The Spectroscope (Methuen, 1951) by permission.

Now I have quoted this extract merely to suggest that the common idea of scientific writing as something essentially bald, direct, and unconcerned with shades of meaning or with images which carry overtones of varying degrees of complexity, is just not true. The scientific writer is often required to go far beyond the mere expression of fact. There are times when he must enlighten, persuade, convince. And there is no rule or formula for superadding 'graces' to your writing. Never try to be 'literary.' The qualities of good writing, clarity of thought, the justness of words, the judgment implicit in imagery, must come from you, and from you alone; and the only way of incorporating them in your own writing is by the enlargement of your sensibility, by your experience of literature of every kind, and (above all) by practice.—T. R. HENN, The Apple and The Spectroscope (Methuen, 1951) by permission.

The Spinning Nucleus in a Magnetic Field: A Source of Classroom Problems

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The object of this paper is to call to the attention of those teaching quantum mechanics several of the problems arising from the behavior of nuclei possessing a magnetic moment in an external magnetic field. These problems are simple enough to serve as student exercises. They are presented both from the classical and quantum-mechanical points of view, and this is particularly instructive in connection with the problems considered because both techniques of calculation lead to the same answers, but the results in spite of formal similarity are to be interpreted quite differently. This provides the teacher with an excellent opportunity to discuss the conceptual differences of the old and new mechanics.

ONE of the difficulties encountered by the teacher of quantum mechanics is that of providing challenging and instructive problems for the members of his class. It is relatively easy to supply the student with numerous mathematical exercises (which exercises are essential, of course, if he is eventually to travel "on his own"), but it is not so easy to confront him with physical problems of interest and importance, and yet of moderate difficulty, so that while lured forward he is not completely bewildered as to how to proceed. As so often happens, the problems that suggest themselves as possible student exercises either prove too difficult or too trivial, and one is constantly searching for situations which will yield to moderate skill and insight, the self-mastery of which will serve to increase both skill and insight. Classical mechanics leaves nothing to be desired in this respect. The almost endless list of problems of all degrees of difficulty enables the instructor to stimulate the minds of his students by posing queries which will cause them to produce their maximum creativeness and ingenuity. But the teacher of quantum mechanics has no such wealth of material at his disposal as was provided by the problems at the end of a chapter of a book on statics or dynamics.

It is the purpose of this article to call attention to the behavior of the spinning nuclear magnet in an external magnetic field, because many of the results obtained could be duplicated by the persevering and creative student working by himself if in the classroom he had been led to grasp the significance of the techniques provided.

Only the simplest situation will be considered, *viz.*, that of a particle of spin quantum number $\frac{1}{2}$ and magnetic moment $\boldsymbol{\mu}$ electromagnetic units. The only field to which it is subjected is $\mathbf{H}$, the externally created field, and apart from this it is isolated from the rest of the universe. This idealized condition is not completely devoid of reality as it describes an environment often encountered in molecular beam experiments. It is instructive in a high degree to employ both the classical and quantum-mechanical techniques in the study of this particle, and in the following this procedure is adopted.

The classical starting point is chosen to be Newton's law which equates the torque on the particle to the change per second of the angular momentum $\mathbf{A}$, giving

$$\frac{d\mathbf{A}}{dt}=(\boldsymbol{\mu}\times\mathbf{H}). \tag{1a}$$

The quantal starting point is Schroedinger's equation

$$\mathcal{H}\Psi=ih\partial\Psi/\partial t, \tag{1b}$$

where $\mathcal{H}=-\mathbf{H}\cdot\boldsymbol{\mu}$ is the classical Hamiltonian for the system, and Ψ, the state function, is a symbolic operand which for a particle of spin $\frac{1}{2}$ must have two components. As the Hamiltonian in its operational form contains the Pauli spin matrices, the Ψ-function will be the column matrix $\begin{vmatrix} c_1 \\ c_2 \end{vmatrix}$, where c_1 and c_2 measure the relative "strengths" of the two states which comprise a given state represented by Ψ. In both problems the choice of axes is completely arbitrary, but in the interests of simplicity they should be related as symmetrically as possible to the external magnetic field which is postulated.

Let us now proceed with a discussion of the classical Eq. (1a). It is known that $|A| = [(1/2)(3/2)]^{\frac{1}{2}}\hbar$, and we will write

$$|\mu| = [(1/2)(3/2)]^{\frac{1}{2}}/\tfrac{1}{2}\gamma\mu_0,$$

where μ_0 is one Bohr nuclear magneton and γ is merely a number characteristic of the nucleus under discussion. Thus $|\mu| = g|A|$ where $g = 2\gamma\mu_0/\hbar$, and since $\mathfrak{y}$ and $\mathbf{A}$ are collinear, Eq. (1a) becomes

$$d\mathfrak{y}/dt = g(\mathfrak{y}\times\mathbf{H}). \tag{2}$$

Equation (2) leads to the following equations for the three components of the vector $d\mathfrak{y}/dt$:

$$\dot{\mu}_x = g(H_z\mu_y - H_y\mu_z), \tag{3a}$$

$$\dot{\mu}_y = g(H_x\mu_z - H_z\mu_x), \tag{3b}$$

$$\dot{\mu}_z = g(H_y\mu_x - H_x\mu_y), \tag{3c}$$

where the dot over a quantity indicates differentiation with respect to the time. If one adds $i(=\sqrt{-1})$ times Eq. (3b) to Eq. (3a) and rearranges Eq. (3c), these equations become

$$\dot{R} + igH_zR = ig(H_x + iH_y)\mu_z, \tag{4a}$$

$$\dot{\mu}_z = \tfrac{1}{2}ig[R(H_x - iH_y) - \bar{R}(H_x + iH_y)], \tag{4b}$$

where $R = \mu_x + i\mu_y$ and a bar over a quantity indicates its complex conjugate. At this point it is necessary to specify the components of the magnetic field and the following have been chosen because they lead to the resonance phenomenon:

$$H_x = H_1\cos\omega t,$$
$$H_y = -H_1\sin\omega t,$$
$$H_z = H_0.$$

This is a constant field in the z direction with a circularly polarized field in the xy plane. Inserting these quantities in Eq. (4) and letting $R = Se^{-i\omega t}$, we find

$$\dot{S} = i\Delta S + i\omega_1\mu_z, \tag{5a}$$

$$\dot{\mu}_z = \tfrac{1}{2}i\omega_1(S - \bar{S}), \tag{5b}$$

where $\Delta = \omega - \omega_0$, $\omega_0 = gH_0$, and $\omega_1 = gH_1$. Now these equations can be solved, subject to any boundary conditions one cares to impose on μ_x, μ_y, μ_z at $t = 0$, but before doing so it is well to notice a particularly simple solution. It is clear that S and μ_z can be independent of the time provided $S = \bar{S}$ and $S = -\omega_1\mu_z/\Delta$. The subsidiary condition $S\bar{S} + \mu_z^2 = \mu^2$, which is equivalent to $\mu_x^2 + \mu_y^2 + \mu_z^2 = \mu^2$, must also be imposed since it is necessary for the three components to add vectorially to give $\mathfrak{y}$. Tracing back through the abbreviations, these equations lead to

$$\mu_x = \frac{\omega_1\mu}{(\omega_1^2 + \Delta^2)^{\frac{1}{2}}}\cos\omega t,$$

$$\mu_y = -\frac{\omega_1\mu}{(\omega_1^2 + \Delta^2)^{\frac{1}{2}}}\sin\omega t,$$

$$\mu_z = \frac{\Delta\mu}{(\omega_1^2 + \Delta^2)^{\frac{1}{2}}}.$$

This is a particularly simple type of motion (precession without nutation) but would probably be unusual because of the small chance of finding a nucleus having the correct initial properties. When $\omega = \omega_0$, we have $\mu_x = \mu\cos\omega t$, $\mu_y = -\mu\sin\omega t$, and $\mu_z = 0$ so that the magnetic moment vector is wholly in the xy plane. It is to be noted that ω_0 is the classical Larmor frequency of precession of the nucleus as a spinning top.

Let us now seek for a solution of Eq. (5) which has the properties that at $t = 0$, $\mu_z = \mu$, and $\mu_x = \mu_y = 0$,—that is, $S = 0$. Differentiating Eq. (5b) twice and introducing $\dot{S}$ and $d\bar{S}/dt$ at each step we obtain

$$d^2\dot{\mu}_z/dt^2 = -\alpha^2\dot{\mu}_z, \tag{6}$$

where $\alpha^2 = \omega_1^2 + \Delta^2$. Thus

$$\mu_z = B\cos(\alpha t + a) + C,$$

where B, a, and C are constants of integration. Substitution of this expression for μ_z in Eq. (5b) gives

$$S - \bar{S} = (2iB\alpha/\omega_1)\sin\alpha t, \tag{7}$$

where the phase angle has been adjusted to make $S - \bar{S}$ vanish at $t = 0$. From Eq. (5a) one gets

$$d(S + \bar{S})/dt = i\Delta(S - \bar{S});$$

that is,

$$S + \bar{S} = (2B\Delta/\omega_1)(\cos\alpha t - 1), \tag{8}$$

where the constant of integration has again been adjusted to make $S + \bar{S}$ vanish at $t = 0$. If the quantities obtained so far are substituted in Eq. (5a), it is found that $C = \Delta^2B/\omega_1^2$. When B is now chosen so as to give μ_z its assumed value at $t = 0$,

one obtains

$$\mu_z=(\mu/\alpha^2)(\Delta^2+\omega_1{}^2\cos\alpha t),$$

$$=\mu\left[1-2\frac{\omega_1{}^2}{\omega_1{}^2+\Delta^2}\sin^2(\tfrac{1}{2}\alpha t)\right]. \quad (9)$$

Note that when $\Delta=0$, that is, when the frequency of the field in the xy plane coincides with the Larmor frequency ω_0, the above formula becomes

$$\mu_z=\mu\cos\omega_1 t, \quad (9a)$$

showing that the z component of the magnetic moment is oscillating back and forth from one extreme position to the other with a frequency determined solely by the amplitude of the circularily polarized field.

It is possible to give Eq. (9) an interesting interpretation which will assume real significance when we come to the quantal consideration of the problem. Suppose for the moment that the value of μ_z is obtained by averaging over two discrete states, one of which has $\mu_z=+\mu$ with a probability of P_1 and the other $\mu_z=-\mu$ with a probability P_2, where $P_1+P_2=1$. Then the average value of μ_z would be given by

$$(\mu_z)_{Av}=\mu P_1+(-\mu)P_2=\mu(1-2P_2).$$

This expression can be identified with Eq. (9) by writing

$$P_2=(\omega_1{}^2/\omega_1{}^2+\Delta^2)\sin^2(\tfrac{1}{2}\alpha t), \quad (10)$$

where P_2 now gives the probability at time t of finding $\mu_z=-\mu$ if at time $t=0$ its value was $+\mu$. This is forcing the classical picture in an unwarranted manner, but it is of interest that Eq. (10) is precisely the answer given by quantum mechanics.

Having completed the above analysis it is now possible to write down

$$\mu_x=\mu\omega_1(\omega_1{}^2+\Delta^2)^{-1}\{-\Delta\cos\omega t + \Delta\cos\omega t\cos\alpha t+\alpha\sin\omega t\sin\alpha t\}, \quad (11a)$$

$$\mu_y=\mu\omega_1(\omega_1{}^2+\Delta^2)^{-1}\{\Delta\sin\omega t - \Delta\sin\omega t\cos\alpha t+\alpha\cos\omega t\sin\alpha t\}. \quad (11b)$$

The first step in the quantal treatment is to express $-\mathbf{H}\cdot\boldsymbol{\mu}$ in its operator form. Since $\boldsymbol{\mu}=g\mathbf{A}$, it may be written as

$$-g\mathbf{H}\cdot\mathbf{A}=-g(H_xA_x+H_yA_y+H_zA_z).$$

The operators to be associated with the x, y, z components of $\mathbf{A}$ are, respectively, $\frac{1}{2}\hbar\sigma_x$, $\frac{1}{2}\hbar\sigma_y$, $\frac{1}{2}\hbar\sigma_z$, where σ_x, σ_y, σ_z are the Pauli spin matrices

$$\sigma_x=\begin{vmatrix}0&1\\1&0\end{vmatrix},\quad \sigma_y=\begin{vmatrix}0&-i\\i&0\end{vmatrix},\quad \sigma_z=\begin{vmatrix}1&0\\0&-1\end{vmatrix},$$

and the operators to be associated with the components of the field are the ordinary algebraic values of these quantities. Thus the equation to be solved is

$$\tfrac{1}{2}ig(H_x\sigma_x+H_y\sigma_y+H_z\sigma_z)\Psi=\partial\Psi/\partial t. \quad (12)$$

Inserting the operators and collecting terms one obtains

$$\begin{vmatrix}\frac{1}{2}ig\{(H_z-iH_y)c_2+H_zc_1\}-\dot{c}_1\\ \frac{1}{2}ig\{(H_x+iH_y)c_1-H_zc_2\}-\dot{c}_2\end{vmatrix}=\begin{vmatrix}0\\0\end{vmatrix}. \quad (13)$$

Since the two entries in this column matrix must each be zero to satisfy the equation, a means has been provided for determining c_1 and c_2. However, as it is only their ratio which has much physical significance, we let $Z=c_1/c_2$, obtaining

$$\dot{Z}=\frac{\dot{c}_1}{c_2}-\frac{c_1}{c_2}\frac{\dot{c}_2}{c_2},$$

$$-2i\dot{Z}=g(H_z-iH_y)+2gH_zZ - g(H_x+iH_y)Z^2. \quad (14)$$

If now the field components have the same values as in the previous case, Eq. (14) becomes

$$-2i\dot{Z}=\omega_1e^{i\omega t}+2\omega_0Z-\omega_1e^{-i\omega t}Z^2, \quad (15)$$

where ω_1 and ω_0 are the same as above. The substitution $Z=fe^{i\omega t}$ yields

$$2i\dot{f}=\omega_1(f^2-1)+2\Delta f, \quad (16)$$

which can readily be integrated giving

$$f=-\frac{\Delta}{\omega_1}+i\frac{\alpha}{\omega_1}\tan(\tfrac{1}{2}\alpha t+b), \quad (17)$$

where b is a constant of integration. It is of interest to note the few number of constants of integration which have appeared in comparison with the number encountered in the classical case. No doubt this follows from the fact that quantum mechanics is not in general expected to predict the precise life history of the particle.

Having obtained a solution of Schroedinger's equation we are now in a position to compute the average values of various dynamical variables by means of the machinery provided by the theory. For example, from the average value of A_z in the state given by the Ψ-function here obtained, one gets

$$(\mu_z)_{Av}=g(A_z)_{Av}=\gamma\mu_0(\sigma_z)_{Av}=\gamma\mu_0\frac{\Phi\sigma_z\Psi}{\Phi\Psi}, \quad (18)$$

where $\Phi=|\bar{c}_1, \bar{c}_2|$ and σ_z is a Pauli matrix. Evaluating Eq. (18) we find

$$(\mu_z)_{Av}=\gamma\mu_0\frac{c_1\bar{c}_1-c_2\bar{c}_2}{c_1\bar{c}_1+c_2\bar{c}_2}$$

$$=(\gamma\mu_0/\alpha^2)[\Delta^2-\omega_1{}^2\cos(\alpha t+2b)]. \quad (19)$$

As yet the Ψ-function has not been made definite because nothing has arisen to make it necessary to fix the quantity b. As it seems to be at loose ends, let us arbitrarily assign it the value $\pi/2$. This makes the Ψ-state unique and formula (19), which becomes

$$(\mu_z)_{Av}=(\gamma\mu_0/\alpha^2)(\Delta^2+\omega_1{}^2\cos\alpha t), \quad (20)$$

enables one to identify the state to the extent that at $t=0$ the z component of μ_z had the value $\gamma\mu_0$. The above expression, it will be observed, is identical with Eq. (9), the classical expression for μ_z, except for the fact that in Eq. (9) there occurs the factor μ while here it is $\gamma\mu_0$. Classically, the magnet can point directly along the z axis while according to the present theory its maximum z component is $\gamma\mu_0$, which is less than μ its total moment in the ratio $\sqrt{3}/3$.

If $b=\pi/2$, the quantity f becomes infinite at $t=0$. Since $c_1/c_2=fe^{i\omega t}$, the logical interpretation is that at this instant $c_2=0$, so that the Ψ-function appropriate to this instantaneous state is

$$\Psi=\begin{vmatrix}c_1\\0\end{vmatrix}.$$

Thus the pure state c_1 is that in which the z component of the magnetic moment has its maximum value $\gamma\mu_0$. Also the pure state c_2 is that in which the z component of μ is $-\gamma\mu_0$. This can be seen most easily by recalling the meaning of $\Phi\Psi=c_1\bar{c}_1+c_2\bar{c}_2$. The quantity $c_2\bar{c}_2$ is the probability (unnormalized) of being in state c_2. For the normalized probability one gets

$$P_2=\frac{c_2\bar{c}_2}{c_1\bar{c}_1+c_2\bar{c}_2}=\frac{1}{1+f\bar{f}}=\frac{\omega_1{}^2}{\omega_1{}^2+\Delta^2}\sin^2(\tfrac{1}{2}\alpha t).$$

Note that this is identical with Eq. (11). At $t=\pi/\alpha$ this probability is a maximum for given Δ, and at this particular time

$$(\mu_z)_{Av}=\gamma\mu_0\frac{\Delta^2-\omega_1{}^2}{\Delta^2+\omega_1{}^2}.$$

Also at $t=\pi/\alpha$ we have $f=-\Delta/\omega_1$, which is not in general equal to zero. Therefore, we may conclude that the particular state we are studying is one which at $t=0$ exists in pure state c_1, and as time passes it tends to go over into pure state c_2 but never quite reaches it unless $\Delta=0$. When $\Delta=0$ we have $f=0$, which means

$$\Psi=\begin{vmatrix}0\\c_2\end{vmatrix},$$

(i.e., pure c_2) and also under these conditions $(\mu_z)_{Av}=-\gamma\mu_0$. Now the two pure states have been identified, and it is clear that when the frequency of the circularly polarized field is equal to ω_0 it is able to induce transitions from one pure state to the other (Cf. Eq. (9a) for the classical prediction).

At this point the teacher has an excellent opportunity to explain to the student the conceptual differences between the two sets of answers which are formally identical. They look alike but they do not "mean" the same thing. For example, classically, if $\Delta\neq0$, the magnet in this particular problem can never point along the negative z axis, while the quantum theory says that it can but with a probability which depends on the time and is always less than unity.

Suppose, for the moment, that the particle is in the steady field H_0 only. When this condition prevails the equation for Ψ is

$$-\gamma\mu_0H_0\sigma_z\Psi=E\Psi, \quad (21)$$

because the Hamiltonian is independent of the time, and a spectrum of energy levels may be expected on searching for Ψ's which satisfy this equation. Equation (21) is equivalent to the two

equations

$$c_1(E+\gamma\mu_0 H_0)=0,$$
$$c_2(E-\gamma\mu_0 H_0)=0,$$

which can obviously be satisfied by having $E_1=\gamma\mu_0 H_0$ and $c_1=0$ simultaneously, or $E_2=-\gamma\mu_0 H_0$ and $c_2=0$ simultaneously. Thus, in the steady field there are two eigenstates associated with the two energy levels given above and according to the Bohr condition the frequency of the radiation which can be absorbed or emitted is given by

$$\hbar\omega=E_1-E_2=2\gamma\mu_0 H,$$

or

$$\omega=\omega_0,$$

the same frequency as was predicted previously for the circular field if it is to induce transitions from one pure state to another. The pure states in the general case also have the same energy as in the case of the eigenstates in the steady field, so that it is possible to say that the rotating field can induce transitions from one eigenstate of the system in the steady field to the other (if it has the right frequency) and that it does not alter the value of these energy levels.

Returning to the problem of the time-dependent field, it is of interest to compute, for example, $(\mu_x)_N$ from the formula

$$(\mu_x)_N=\gamma\mu_0\frac{\Phi\sigma_x\Psi}{\Phi\Psi}=\gamma\mu_0\frac{fe^{i\omega t}+\bar{f}e^{-i\omega t}}{1+f\bar{f}},$$

and obtain the same result as is given in Eq. (10a) except for the quantity μ which becomes $\gamma\mu_0$.

The time spent by a student attempting to work his way through problems such as those presented above would surely enhance his appreciation of the theory of the subject. If he has reached the stage where he is beginning to appreciate the methodology of quantum mechanics, it is important for the teacher to give him something on which to try his skill.

For the student of exceptional ability one can set the problem of predicting the behavior of a particle in an elliptically polarized field. This, as might be expected, is a much more troublesome situation, but it is important for the student to learn that by making judicious approximations it is usually possible to win through to a more or less satisfactory answer. A brief discussion of this more general case from the point of view of quantum mechanics will now be presented.

The following components are adopted for the magnetic field:

$$H_x=H_1\cos(\omega t+\phi_1),$$
$$H_y=H_2\cos(\omega t+\phi_2),$$
$$H_z=H_0.$$

These values are to be inserted into Eq. (14), and for this purpose it is possible to write

$$H_x+iH_y=Pe^{i\omega t}+Qe^{-i\omega t},$$
$$H_x-iH_y=\bar{P}e^{-i\omega t}+\bar{Q}e^{i\omega t},$$

where

$$2P=H_1e^{i\phi_1}+iH_2e^{i\phi_2}, \tag{22a}$$

$$2Q=H_1e^{-i\phi_1}+iH_2e^{-i\phi_2}. \tag{22b}$$

Adopting the abbreviations $gQ=\omega_1$, $gP=\omega_2$, and $gH_0=\omega_0$ (ω_1 and ω_2 are now complex) Eq. (14) becomes

$$-2i\dot{Z}=(\bar{\omega}_2e^{-i\omega t}+\bar{\omega}_1e^{i\omega t})+2\omega_0 Z - (\omega_2e^{i\omega t}+\omega_1e^{-i\omega t})Z^2. \tag{23}$$

Let $Z=fe^{i\omega t}$, giving

$$-2i\dot{f}=(\bar{\omega}_1+\bar{\omega}_2e^{-2i\omega t})-2\Delta f-(\omega_1+\omega_2e^{2i\omega t})f^2. \tag{24}$$

It is at this point that the need for caution arises because the above equation, as it stands, is insoluble. An examination of the definitions of ω_1 and ω_2 shows that if the rotating field is circularly polarized (i.e., if $H_1=H_2$ and $\phi_2-\phi_1=\pi/2$), the quantity ω_2 is zero and that if the field is only slightly elliptical, $|\omega_2|$ is still much smaller than $|\omega_1|$. As ω_2 is multiplied by a rapidly oscillating quantity (of modulus unity) so that this small term averages out to zero, one might consider a useful first approximation to be that in which the terms in ω_2 are dropped. If this be done Eq. (24) is readily integrated and the result obtained is the same as before (see Eq. (17)) except that ω_1 (which was real in the simpler case) must always be replaced by $\bar{\omega}_1$ when it stands by itself and by $\omega_1\bar{\omega}_1$ wherever a squared term appears. Thus we have obtained the solution for an elliptical field of low eccentricity (and $\phi_2-\phi_1\simeq\pi/2$) and find nothing changed except ω_1, and in particular the frequency at which resonance occurs is unaffected (to this degree of approximation) and remains ω_0.

However, it is possible to let the approximation

cover a wider range by the substitution

$$f=g-(\bar{\omega}_2/4\omega)e^{-2i\omega t},$$

which leads to the following equation for g:

$$-2i\dot{g}=\bar{\omega}_1-2g\left(\Delta-\frac{\omega_2\bar{\omega}_2}{4\omega}-\frac{\omega_1\bar{\omega}_2}{4\omega}e^{-2i\omega t}\right)-(\omega_1+\omega_2e^{2i\omega t})g^2, \quad (25)$$

in which terms of order $|\omega_2/\omega|$ have been dropped in comparison with terms of order unity. This means that the fields in the xy plane must be small in comparison with H_0. Two further changes of dependent variable are required. First let $g=1/h$ and then let

$$h=r+(\omega_2/4\omega)e^{2i\omega t},$$

and again drop terms of order $|\omega_2/\omega|$ in comparison with unity. The equation for r is

$$-2i\dot{r}=\omega_1-\bar{\omega}_1r^2+2\left[\Delta-\frac{\omega_2\bar{\omega}_2}{4\omega}-\frac{1}{4\omega}(\bar{\omega}_1\omega_2e^{2i\omega t}+\omega_1\bar{\omega}_2e^{-2i\omega t})\right]. \quad (26)$$

Equation (26) is still insoluble because of the time dependence of the term multiplying r. However, if one permits only those fields which have $\phi_2-\phi\approx\pi/2$, ω_2 is quite small even for considerable difference between H_1 and H_2. Furthermore, the terms containing ω_2 are two orders of magnitude smaller than ω or ω_0 with which they occur, so that their presence in this term is of slight consequence except right at resonance. One might be tempted to drop both of them, but as dropping the time-dependent one, which rapidly averages out to zero, makes the equation soluble, it alone will be discarded. Then r is given by

$$-2i\dot{r}=\omega_1-\bar{\omega}_1r^2+2\delta r, \quad (27)$$

where

$$\delta=\Delta-\omega_2\bar{\omega}_2/4\omega.$$

Solving this, we obtain

$$r=\frac{\delta}{\bar{\omega}_1}+i\frac{\beta}{\bar{\omega}_1}\tan(\tfrac{1}{2}\beta t), \quad (28)$$

where

$$\beta^2=\omega_1\bar{\omega}_1+\delta^2,$$

the constant of integration being set at zero arbitrarily to simplify the mathematics. If one traces the process back through the various changes of variables, making the legitimate approximations at each stage, one is now able to deduce that by far the dominant term in the formula for the probability of a state is

$$\frac{\omega_1\bar{\omega}_1}{(\omega_1\bar{\omega}_1+\delta^2)^{\frac{1}{2}}}\sin^2\tfrac{1}{2}\beta t,$$

and this formula is of the same form as Eq. (11) except for the fact that the various quantities show a dependence on the degree of eccentricity of the polarized field. The most important feature is the prediction of a slight change in the frequency at which resonance takes place. This probability function has unit amplitude (this means complete passage from one state to the other) when

$$\delta=\omega\left(1-\frac{\omega_2\bar{\omega}_2}{4\omega^2}\right)-\omega_0=0,$$

where

$$\left(1-\frac{\omega_2\bar{\omega}_2}{4\omega^2}\right)\simeq 1-\frac{1}{16}\frac{H_1^2+H_2^2+2H_1H_2\sin(\phi_2-\phi_1)}{H_0^2}.$$

Enough has been written to indicate the high degree of usefuness of this branch of quantum mechanics, which is of vital current interest, as problem material for the student. It has not seemed necessary to give references to the current literature as all the problems discussed are simple applications of techniques described in intermediate textbooks on quantum mechanics. By way of general reference it may be said that most of the above considerations can be found (but in considerably different form) in the writings of those engaged in molecular beam or nuclear induction work.

The stimulus for the above study was provided by a grant from the National Research Council of Canada for the purpose of investigating nuclear magnetic moments.

NOTES AND DISCUSSION

An Easily Constructed Alpha-Particle Range Apparatus

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IN our undergraduate laboratory in atomic and nuclear physics we have included an experiment on the range of alpha-particles in air and the stopping power of thin metal foils by the scintillation method. This experiment is included, not because of its accuracy when performed by the average student, but because of its historical relation to the great pioneering work of Rutherford. The apparatus used is patterned after a form suggested by Hoag and Korff[1] and is shown in Fig. 1. We found that we had a number of vernier calipers whose jaws no longer closed with sufficient precision to be useful as measuring instruments in the laboratory but seemed ideally suited for adaptation to an alpha-particle range apparatus.

The alpha-particle source consists of discarded radon in gold needles commonly used in radiotherapy. The gold needles fit into small holes drilled in a brass cylinder fastened to the movable jaw of the vernier caliper. The gold needles are cut without pinching them off by placing them on a smooth flat surface and running a sharp razor blade across them in a gentle rolling motion. The lengths are carefully adjusted so that the open ends of the needles are flush with the top when inserted in the holes in the brass cylinder.

The eyepiece tube fastened to the fixed jaw of the caliper was bored on a lathe to an internal diameter to give a snug fit to the mounting tube of a simple magnifier having a focal length of about 2.5 cm. These simple magnifiers were obtained in surplus for a few cents each. An eyepiece from an ordinary laboratory telescope should be as satisfactory. A small shoulder was turned in the lower end of the eyepiece tube with a diameter just sufficient to accomodate a small glass plate. These thin circular glass plates were obtained from the biology laboratory.

FIG. 1. Photograph of the alpha-particle range apparatus.

The cylindrical source and eyepiece tubes are fastened to the caliper jaws with soft solder. A small vee groove filed in the face of the jaws is helpful in aligning the two pieces during the soldering process.

The holder for the thin metal foils used in stopping-power measurements consists of two thin sheets of brass about 3×4 cm with circular holes cut in them the size of the end of the source cylinder. These sheets are spaced about 1 mm apart by inserting them in slots cut in a small metal block fastened to the caliper by means of a small machine screw. The sheets are soldered to the block after proper alignment with respect to the source and screen (see Fig. 1).

The caliper assembly is mounted on a wooden base by means of a machine screw passing through two small metal angles. A wing nut is used for clamping and ease of changing the angle of the instrument for comfortable viewing. The circular glass plate is waxed into position and covered with a thin film of transparent cement. Some phosphorescent zinc sulfide powder is sprinkled on the cement and when dry the excess is shaken off, leaving a thin coating of sensitive material. Since the zinc sulfide is sensitive to ordinary light and continues to glow for a considerable time after it is irradiated, it is necessary to have a cover for the instrument which can be readily removed after the student has entered the darkroom to begin the experiment. An ordinary cylindrical paper carton used originally for packaging oatmeal serves very well. The top of the carton is coated with a phosphorescent paint so that the student can find the instrument in the dark. The thin foils used for stopping-power measurements are mounted between two sheets of thin cardboard containing circular holes which match those in the thin brass sheets. Their appearance is similar to the half-wave plates found in the optics laboratory.

The procedure followed in the experiment is that outlined by Hoag and Korff,[1] Experiment 12–3. An ordinary darkroom safelight with a red filter is used for illuminating the scale while readings are being recorded. This gives sufficient light and does not delay the time between readings to any great extent. If students work in pairs, it is usually faster for one of them to make the settings and have the other record the readings at the safelight a few feet away since the instrument is easily handed back and forth.

The values obtained with this apparatus for the mean range of alpha-particles from polonium usually agree with the accepted value of 3.83 cm to within 10 percent. The agreement with the accepted values for the stopping power of aluminum foil is less satisfactory, often differing as much as 50 percent. Regardless of the inaccuracies encountered all of the students come out of the experience with a greater admiration and respect for the discoveries of Rutherford unveiled by the use of the scintillation technique.

[1] J. Hoag and S. Korff, *Electron and Nuclear Physics* (D. Van Nostrand Company Inc., New York, 1948), p. 298.

A Wire Model for the Law of Reflection

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THE law of reflection is apparently a simple and straightforward matter and usually receives only brief discussion in an elementary physics course, yet there are some features of the description of what happens to a ray upon striking a reflecting surface that are worthy of more than passing consideration. In order to facilitate the discussion, the simple model shown in the figure was constructed and is exhibited during the introductory demonstration lecture on ray optics.

The "normal" is made of $\frac{1}{4}$-inch brass tubing, the major portion being fixed to an internal rod mounted on a plywood base. The incident ray is rigidly attached at the lower end, while the reflected ray is soldered to the short portion of the tube which is rotatable. Both rays are made of $\frac{1}{8}$-inch brass rod. Each ray is equipped with an indicator showing the angle of incidence or of reflection, as the case may be, and sheet metal arrows show the direction in which the light travels. The upper surface of the base may be given a coat of aluminum paint to suggest a mirror. Overall height of the model is about 11 inches.

The model provides an opportunity for answering the inevitable question: "Why are the angles specified with respect to the perpendicular rather than with the surface itself?" If desired, the instructor may emphasize the fact that the base is in general to be considered a surface element of the reflector, which may not even be plane. But perhaps the greatest utility of the model with its rotatable reflected ray lies in the possibility of showing that, in addition to equality of the two angles, the reflected ray must lie in the plane determined by the incident ray and the normal. As a matter of fact, inspection of elementary textbooks shows that some actually fail to state this essential part of the principle—a feature which seems perfectly reasonable to the student once it is pointed out. Inasmuch as beginning students are not too facile in visualizing geometric relationships in space, the model helps materially toward making this point clear.

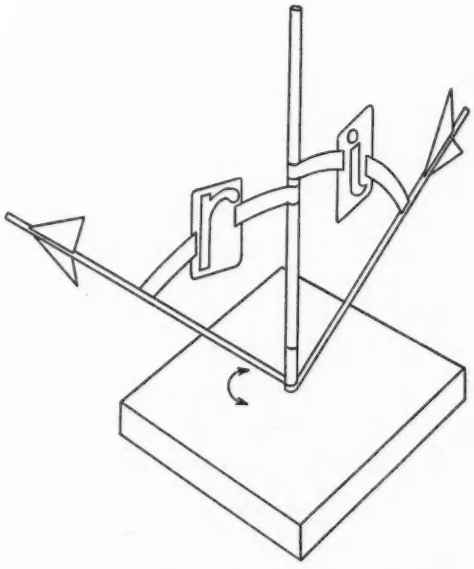

FIG. 1. Wire model to illustrate the law of reflection.

Mr. E. Wilson of our shop staff kindly helped with the design and construction of the model.

On Fields and Lines of Force

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IT would certainly be difficult to discuss the effects of electric and magnetic fields at the elementary level without introducing Michael Faraday's concept of the line of force. The method, however, has its limitations and pitfalls.

1. The line of force is a dichotomous concept in the following respect. On the one hand, we may define it as the locus of a series of connected infinitesimal vectors pointing in the direction of the field; or, to use the terminology that we give our students, " . . . an imaginary line drawn in such a way that its direction at any point is the same as the direction of the field at that point." [1] (Here we are taking the liberty of giving a "line" direction!) Perhaps a better definition is ". . . a line whose tangent at each point is the direction of the (electric) intensity vector at that point."[2] The lines of force in a field should be *infinitely dense*, as certainly there is a force on a (generalized) charge at each point, excepting, of course, in any regions where the field happens to be zero. On the other hand, we use the *density* of lines of force as a measure of field intensity.[3]

2. The student is liable to get the idea that the line of force actually exists. This misconception is heightened by such statements as "the lines pull unlike (magnet) poles together" (actually seen on a slide made by a well-known scientific company). It should be emphasized that the line of force is merely a device to help understand the actual physical situation.

3. The line of force is by no means the line along which a (generalized) charge will move, as at least one standard text remarks.[4] Only if the body somehow moves quasi-statically would it even *tend* to move along a line of force. In electrostatics, a thorough discussion of this point would help to lay the groundwork for the understanding of the difficult idea of potential; since the field is conservative, we have an energy balance among the potential energy of the charge in the field, any external work done by or on the charge, and the change in kinetic energy of the charge. The student often has difficulty in understanding what we mean by the statement "the field does work on the charge"; this work generally appears as an increase in kinetic energy.

4. It is awkward to introduce the free north pole just in order that it may experience a force in the direction of the magnetic field vector, and then deny its existence altogether. If magnetostatics is to be introduced before the magnetic effects of currents—and the merit of this

is not the point under discussion here—it is best to say that the magnetic field vectors, out of which are composed the lines of force, point in the direction along which an infinitesimal magnetic dipole would be oriented. This viewpoint also ties in better with the laboratory experiment on mapping a magnetic field with a small compass.

5. It has been my experience that introduction of the concept of the gravitational field and gravitational lines of force in order to help the student understand electric and magnetic fields has not been singularly successful.

[1] F. W. Sears, *Principles of Physics* (Addison-Wesley, Inc., Cambridge, 1946), Vol. II, p. 32.
[2] Shortley and Williams, *Physics* (Prentice-Hall, Inc., New York, 1950), Vol. II, p. 799.
[3] It is to be noted that these two lately published texts (references 1, 2) seem to be the only ones that remark on this duality of the line of force concept. This could be attributed to the fact that they are on a more advanced level than most other beginning physics textbooks.
[4] E. Hausmann and E. P. Slack, *Physics* (D. Van Nostrand Company, Inc., New York, 1939), p. 342.

An Experiment on Torque

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FOR some time we have desired a simple torque experiment which used neither paper,[1] special torque bars,[2] nor lever arms already perpendicular to forces.[3] Recently we devised the simple structure shown in Fig. 1 using three meter sticks, three lever clamps, and three plumb bobs. Results with this arrangement have been very satisfactory.

Two one-meter sticks and one half-meter stick have holes bored at appropriate points to enable them to be held as a 30–60–90 right triangle. It was found that pins made from the head end of number 10 nails worked perfectly. On each meter stick a lever clamp[4] is placed. Each clamp may have hung on it a thread with a washer to act as a plumb bob.

The structure is hung from any one of the clamps and the attached stick made horizontal to the eye by sliding it in the clamp. (Any orientation is allowed, but students feel better with horizontal positions.) The plumb-bob thread is then fastened to the lower sticks at the point of crossing with some Scotch tape. By repeating the process with each side in turn, the student locates the center of mass as the intersection of the three threads. The third thread intersects the other two over a range of 0.5 cm. The fact that the third thread crosses the intersection of the first two is a source of considerable amazement to most students.

The student then checks the second condition of equilibrium by hanging an arbitrary weight (usually 200 to 500 g) from one of the two lower clamps. He measures the lever arms about the point of support by hanging a fourth plumb bob from it and using a half-meter stick along with "the shortest distance" definition of a perpendicular. In spite of the apparent crudity of the lever arm measurements, students are generally within one percent in comparing positive and negative torques.

The advantages of this type of experiment are threefold. First, no clues are left for the next victims because removal of the Scotch tape and displacement of the clamps leaves the structure in its original state. Second, the student learns how to locate the center of mass and sees that it need not be in the actual mass of the object. Third, the necessity for measuring lever arms as perpendiculars to force directions is obvious, for no other measurement will check.

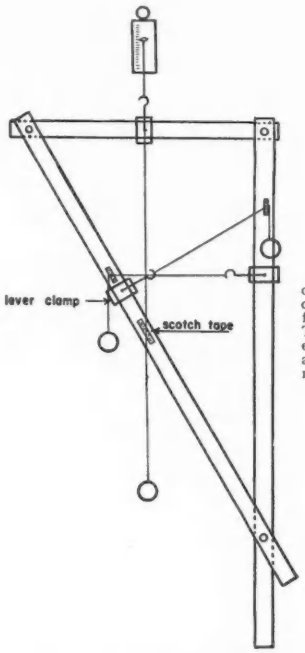

FIG. 1. Pinned structure of meter sticks with lever clamps and plumb bobs for locating center of mass. The plumb bobs are washers, of which the masses are, relatively speaking, neglible.

A fourth point is actually emphasized in the experiment of which the above is just one part. In all cases the support of the torque experiment is a spring balance which reads the total weight of the supported objects, said objects having previously been weighed separately on a trip scale. The fact that the balance reads the total weight of all supported objects still seems to amaze many students in spite of repeated class discussions of the first condition of equilibrium.

[1] D. A. Woodbury and C. W. Jarvis, *Laboratory Manual for Arts Students* (R. G. Adams & Company, Columbus, Ohio, 1936), fifth edition, p. 87.
[2] L. W. Taylor, W. W. Watson, and C. E. Howe, *General Physics for the Laboratory* (Ginn and Company, New York, 1942), revised edition, p. 7.
[3] B. Cioffari, *Experiments in College Physics* (D. C. Heath & Company, New York, 1951), p. 15.
[4] Central Scientific Company, Catalog J 141, No. 75555, p. 1101.

An Electrical Method for Solving the Lens Equation

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IN these days of analog and digital computers there is considerable student interest in electrical and mechanical mathematical machines. However, many of the methods

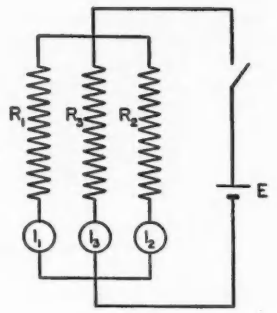

FIG. 1. Basic electrical circuit for solving the lens equation.

employed in such devices are beyond the knowledge of the elementary physics student. But there are some simple circuits which can be used to demonstrate mathematical devices. One example is the Wheatstone bridge method for extracting square roots.[1]

The purpose of this note is to illustrate another example. The equation

$$\frac{1}{x}+\frac{1}{y}=\frac{1}{z} \tag{1}$$

is encountered in many physical phenomena, e.g., the lens equation, resistors in parallel, and condensers in series. The basic circuit for its solution is shown in Fig. 1. The currents i_1, i_2, and i_3 represent the variables x, y, and z,

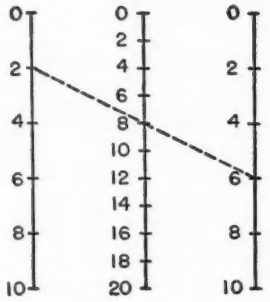

FIG. 2. Nomograph for adding two numbers.

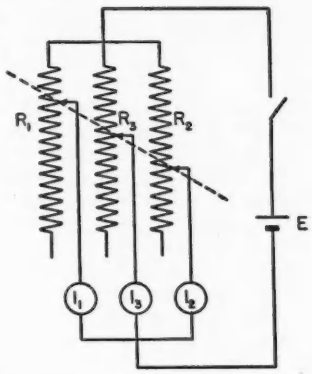

FIG. 3. Final form of the electrical circuit for solving the equation.

respectively. These currents are given by Ohm's Law:

$$i_1=\frac{E}{R_1}, \qquad i_2=\frac{E}{R_2}, \qquad i_3=\frac{E}{R_3}.$$

When these values are substituted for the currents in Eq. (1), it is found that

$$R_1+R_2=R_3. \tag{2}$$

Thus, when the conditions of Eq. (2) are satisfied, the currents represent the variables x, y, and z.

A mechanical method of satisfying Eq. (2) employs nomographic principles. Figure 2 shows a nomograph which can be used. When a straight line is drawn so that it intersects the three vertical axes, the reading of its intersection on the middle line is the sum of the intersections on the two outer lines provided the inner axis has twice as many units as the other two and provided it is located midway between them.

In Fig. 3 this nomographic property is employed using resistors. The inner resistance is twice that of the outer ones. The sliding contacts on the resistors are kept in a straight line by means of a straight bar connecting them and Eq. (2) is thus satisfied.

[1] W. K. Ergen, Rev. Sci. Instr. **18**, 564 (1947).

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LETTERS TO THE EDITOR

When a Body Falls the Earth Turns Faster

PROFESSOR Satterly's recent note[1] on the value of g raises several interesting points. Since the work done in lifting a mass m to a height h is mgh, the heat released when it falls and strikes the earth must certainly be mgh, but the total loss in gravitational potential is $m\gamma h$, where γ is the acceleration of gravity on a nonrotating earth and $g<\gamma$ except at the poles. What happens to this energy difference $(m\gamma h-mgh)$, which is about 0.3 percent of mgh at the equator?

Consider the body of mass m falling from a height h at the equator to make an inelastic impact with the earth. Let R be the radius of the earth, I its moment of inertia, and ω its angular velocity. When the mass moves from its initial fixed position at a distance $(R+h)$ from the earth's axis to its final fixed position at a distance R from the axis, the change in the total moment of inertia is given by

$\Delta I = mR^2 - m(R+h)^2 = -2mRh$, neglecting the second-order term. Observed from a nonrotating reference frame, the angular momentum of the system is conserved, i.e., $I\omega$ = a constant and hence the change in angular velocity can readily be determined: $\Delta\omega = (-\omega/I)\Delta I = +2m\omega Rh/I$. Thus when a body falls the earth's rotational speed is increased. The phenomenon is similar to the figure-skater's accelerated spin. Naturally the effect on the earth is very small. A 10-kg mass falling 10 meters causes the length of the day to be shortened by about 10^{-18} microsecond.

The increase in rotational kinetic energy of the system, which results from the change in angular velocity, is easily calculated: $\Delta(\frac{1}{2}I\omega^2) = I\omega\Delta\omega + \frac{1}{2}\omega^2\Delta I = m\omega^2 Rh$. This energy, of course, comes from the gravitational field which is causing the body to fall. Since the total loss in gravitational potential energy is $m\gamma h$ as mentioned previously, the energy which must be released as heat on impact, and hence the kinetic energy of the mass (relative to the earth) just prior to impact, is the loss in gravitational potential energy minus the gain in rotational kinetic energy, $m\gamma h - m\omega^2 Rh = m(\gamma - \omega^2 R)h$. But the acceleration of gravity g at the equator is $(\gamma - \omega^2 R)$, so that the heat released is mgh, as anticipated. Thus it is clear that the energy $(m\gamma h - mgh)$ is added to the rotational kinetic energy of the system, and so for a body falling at the equator about 0.3 percent of the loss in gravitational potential energy appears as an increase in the rotational kinetic energy of the earth.

Another approach, still employing conservation of angular momentum, is as follows. The velocity of the body is not precisely in a vertical direction, but has the well-known eastward component relative to the earth, which can be calculated in the following way. Since the gravitational attraction is a central force, the angular momentum of the mass itself is conserved during the fall. Hence $m\omega(R+h)^2 = m\omega' R^2$, where ω' is the new angular velocity of the body just before impact. The horizontal component of the velocity of the body relative to the earth is therefore $(\omega' - \omega)R = 2\omega h$, neglecting the term in h^2. On impact the angular momentum transferred to the earth is $m \times 2\omega h \times R$, and thus the increase in its angular velocity is $2m\omega hR/I$, as before.

Next consider the phenomenon from the viewpoint of an observer on the earth. In a rotating reference frame it is customary to introduce the non-Newtonian, and therefore sometimes called fictitious, centrifugal, and Coriolis forces. The increase in the rotational kinetic energy, $m\omega^2 Rh$, can be regarded as the work done by the gravitational attraction against the centrifugal force $m\omega^2 R$. The eastward component of the velocity, $2\omega h$, is the result of the horizontal Coriolis force. The kinetic energy associated with this horizontal velocity, *viz.*, $\frac{1}{2}m(2\omega h)^2$, is provided for by the loss in gravitational potential, since the horizontal motion involves a vertical Coriolis component which adds to the centrifugal force. A simple calculation shows that the work done by the gravitational attraction against this vertical Coriolis force just equals $\frac{1}{2}m(2\omega h)^2$. At the same time it is clear that the vertical component of velocity for a freely falling body is slightly less than the usual $(2gh)^{\frac{1}{2}}$—it is $(2gh - 4\omega^2 h^2)^{\frac{1}{2}}$. The total velocity will be, of course, $(2gh)^{\frac{1}{2}}$, since the kinetic energy is always mgh. If the body were constrained to move in a vertical line (say by sliding down a slippery vertical pole) the vertical component would necessarily be $(2gh)^{\frac{1}{2}}$. For this case it is the continual action of the horizontal Coriolis component (against the pole) during the fall which would give the increased rotational speed to the earth, rather than the angular momentum transferred at impact as in the free fall.

It can be pointed out in conclusion that the use of the appropriate Newtonian reference frame always gives the most direct solution to such problems and the use of rotating frames adds complexities without introducing any new features. However, since man does make his observations from the rotating earth, it is sometimes instructive to consider the phenomenon from this viewpoint.

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[1] John Satterly, Am. J. Phys. **19**, 322 (1951).

An Improved Mechanical Equivalent of Heat Experiment

IT has been brought to my attention that there is an omission in my recent article "An Improved Mechanical Equivalent of Heat Experiment."[1]

In footnote 3, a method was suggested for converting a single-phase induction motor so that it would run synchronously.

It should be added that, after making the four saw cuts in the rotor, it is necessary to reconnect the end where they have been severed by the hack saw. For ease in soldering, one should, of course, select a motor having a copper conductor grid, rather than aluminum. Also, one should be careful not to cut through any of the conducting bars, but should cut between them in the case of a rotor which has the spiral structure.

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[1] G. G. Kretschmar, Am. J. Phys. **19**, 509 (1951).

Character in Teaching

MR. HARRY PEACH'S letter "Short Laboratory Manuals are Efficient,"[1] invites the comment that almost every paper published in the *Journal* begins with a well-composed abstract, then tells its story concisely, and finally puts it across in a summary. Mr. Peach proposes that laboratory directions be introduced by abstracts, and there are the best of precedents for such a practice. Mr. Peach laments the fog of words that enshrouds so many manuals. Was not the remedy suggested long ago by an editor of *The Physical Review* who predicted that ninety-nine percent of controversies would vanish if every contributor were required to master English composition in addition to physics?

One feature of Mr. Peach's letter calls for special mention. The entire letter reveals that its writer is deliberately

at work trying to make the work of his pupils in the laboratory a more genuine, ethical performance of higher educational value. He is not neglecting his pupils with some noble aim in view. He is not waiting for more favorable circumstances—a larger budget, smaller sections, new equipment, a more commodious laboratory. He is at work *now*.

As a young man the writer had the good fortune to be acquainted with and to feel the inspiration of the late Provost Edgar Fahs Smith.[2] While serving as vice-provost of a large university, director of a large laboratory, and chairman of a large department, while president of the American Philosophical Society and a member of the National Academy of Sciences, and after serving the first of his three terms as president of the American Chemical Society, and while active in scientific, educational, and public projects of many kinds, Dr. Smith was able to keep in close touch with us undergraduates in the laboratories, to visit us regularly, to assure himself that our work was progressing as it should.

Edgar Fahs Smith had neither the strength of an ox nor superhuman intelligence, but he did have CHARACTER. Warner and Swasey advertise that

> There is nothing the matter with this country that *character* won't solve.

This includes whatever is wrong with laboratory instruction. Only intelligence and character in partnership will set it right.

When selecting personnel to teach our undergraduates in the laboratories, let us continue to count up the courses they have taken, the degrees they hold, the papers they have published, and besides let us evaluate how much working character they possess.

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[1] H. Peach, Am. J. Phys. **20**, 54 (1952).
[2] T. D. Cope, "Cultural by-products of laboratory instruction," J. Chem. Educ. **24**, 193–4 (1947).

Another Fluid Paradox

AN interesting paradox in perfect fluid theory is that of the motion of a rigid, free, spherical obstacle in a moving perfect fluid. It is rather carefully discussed in Lamb[1] in the scattering of an incident plane wave on a free, spherical obstacle. As is to be expected, the motion of the object vanishes if its density is infinite compared to the fluid, the motion just equals that of the fluid if their densities just match, but the paradox is that its acceleration is just *three* times that of the fluid if its density vanishes. (It is assumed that the local wavelength is large compared to the radius of the sphere.)

We can rederive this result on a simpler and more intuitive basis by considering a frame of reference which is accelerating with the fluid (which we assume is in rectilinear motion with coordinate ζ_0 at the position of the center of the sphere). In this frame of reference the local g is $-\partial^2\zeta_0/\partial t^2$. Therefore, in this frame, the "buoyant" force acting on the sphere is $-(M-m)\partial^2\zeta_0/\partial t^2$, where M is the mass of the sphere and m is the mass of the displaced water. We at once apply Newton's law to the motion of the sphere but we must remember that the effective mass of the sphere is $M+\frac{1}{2}m$, the well-known additional term arising from the acceleration of the displaced water. The resultant equation (in the accelerating frame of reference) is

$$-(M-m)\partial^2\zeta_0/\partial t^2=(M+\tfrac{1}{2}m)(\partial^2\zeta/\partial t^2-\partial^2\zeta_0/\partial t^2), \quad (1)$$

where ζ is the absolute coordinate of the center of the sphere and therefore $\partial^2\zeta/\partial t^2-\partial^2\zeta_0/\partial t^2$ is the relative acceleration. Solving for $\partial^2\zeta/\partial t^2$ gives

$$\partial^2\zeta/\partial t^2=\frac{3}{1+2\rho/\rho_0}\partial^2\zeta_0/\partial t^2, \quad (2)$$

where ρ and ρ_0 are the densities of the ball and the fluid, respectively.

In the special case of an incident sound wave whose wavelength is long compared to the sphere, the motion of the fluid is sinusoidal with time. The motion of the sphere and fluid will be in phase, and the ratio of the amplitudes is $3/(1+2\rho/\rho_0)$. The constants of integration of Eq. (2) present no problem and may be set equal to zero.

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[1] H. Lamb, *Hydrodynamics* (Cambridge University Press, New York, 1932), sixth edition, p. 514.

Replies to Inquiring Letters

IN reply to J. S. Miller's inquiry[1] about the experimental pressure relations in an inflated rubber balloon, may I say that we have carried out such experiments in our classes for many years. We use what is known in the trade as a 9-in. balloon; and the stopper closing the neck of the balloon carries tubes connecting to the balloon, to the air supply, and to a U-tube water manometer, respectively. At intervals during inflation the air supply is turned off, the diameter of the balloon measured with an improvised pair of calipers, and the excess air pressure read on the manometer.

Table I shows a typical set of observations taken from the notebooks of a pair of students.

If the balloon does not burst, readings may be taken as the balloon is deflated; the results are similar but for the same values of R the values of p are much lower. Thus we see that as the balloon is inflated there is a quick rise of p to a maximum followed by a gradual fall to a minimum and a further slight rise before the balloon bursts. If a graph is drawn between R as abscissa and p as ordinate,

TABLE I. Observations on balloon pressure.

Radius R (cm)	Pressure excess p (cm of water)	Rp
11.8	5.0	59
14.2	18.4	261
17.9	17.8	318
20.8	15.7	327
26.2	12.8	335
36.2	9.7	351
44.8	8.7	390
50.8	8.8	447
55.6	9.5	528
64.8	balloon burst	——

it is possible to draw a line at constant pressure to cut the graph in three points, giving the three values of R for the same value of p.

When I first did this experiment, I was surprised with the result, and one reason for incorporating it into class use was to enable the student to contrast the behavior of soap bubbles and rubber balloons. We hoped also to get sufficient data to calculate the Young's modulus of rubber, but this project has not proved very successful—and there are far better ways of getting the elastic moduli.

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[1] Am. J. Phys. **20**, 115 (1952).

Experiment on Positron Annihilation

IN the February number of this Journal, Professor J. S. Levinger[1] described an admirable lecture table demonstration of positron annihilation, quite appropriate for undergraduate courses. In addition to its intrinsic interest this experiment provides an excellent illustration of the photon nature of radiation. It would seem, however, that the role of Einstein's mass-energy relation might have been more explicitly emphasized, since it provides the only available method for measuring the total energy resident in any material particle.

Before the discovery of the positron, there was considerable speculation regarding the possibility of utilizing the intrinsic rest-energy of matter, since it is very natural to assume that energy in one form may be converted into energy in other forms. If this should occur, the law of energy conservation would imply a conservation of mass also, since $E=mc^2$. The idea of the conservation of matter, however, would be invalidated.

In pair production and annihilation the emphasis should be upon the conversion of matter into radiation, neither of which is conserved. Thus, in annihilation the total energy of the two particles, which may be written $m_1c^2+m_2c^2$, is transformed into the energy of two photons, $h\nu_1$ and $h\nu_2$. Since this usually occurs when the velocities are small, the total momentum is zero, and to a good approximation $m_1=m_2=m_0$, where m_0 is the rest mass of the electron. Since the sum of the two photon momenta must also be zero, they are opposite in sign and $h\nu_1/c=h\nu_2/c=m_\phi c$, where m_ϕ is the mass carried away by each photon, and is equal to m_0.

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[1] Am. J. Phys. **20**, 71 (1952).

Errata: Undergraduate Origins of American Physicists

AS a consequence of a regrettable error in which a list of partial tabulations was used in place of the correct totals, a number of excellent institutions were not given proper recognition for their contributions in training undergraduate students in physics. Table I is a corrected presentation of the number of physicists in the 1949 edition of *American Men of Science* who graduated from each of the listed institutions and of the "productivity index" obtained by dividing the number of graduates by the number of students in thousands enrolled in 1935–1936.

TABLE I. The number of physicists in the 1949 edition of *American Men of Science* who received undergraduate degrees from the institutions listed and the number per thousand students enrolled in 1935–1936 for these institutions.

Institution	Location	No.	No. per thousand
Oberlin	Ohio	52	31
Miami	Ohio	32	12
Ripon	Wisconsin	28	85
Reed	Oregon	24	53
Utah	Utah	22	6
Brigham Young	Utah	21	9
De Pauw	Indiana	21	16
Wesleyan	Connecticut	20	30
Denison	Ohio	19	23
Pomona	California	19	24
Colorado College	Colorado	17	29
Park	Missouri	16	31
St. Olaf	Minnesota	16	16
Wooster	Ohio	16	17
Amherst	Massachusetts	15	18
Buffalo	New York	15	9
Carleton	Minnesota	15	21
Davidson	N. Carolina	15	23
Haverford	Pennsylvania	14	43
Whitman	Washington	14	22
Cornell College	Iowa	13	23
Ohio Wesleyan	Ohio	13	9
Richmond	Virginia	13	13
Utah State	Utah	12	5
Denver	Colorado	11	3
Georgia	Georgia	11	4
Temple	Pennsylvania	11	2
Williams	Massachusetts	11	13
Washington State	Washington	11	3
Colgate	New York	10	10
Lafayette	Pennsylvania	10	12
Western Reserve	Ohio	10	2
William and Mary	Virginia	10	8

Knox College is listed twice in Table II of the paper; the first listing (productivity index=22.6) is the correct one. The number of graduates of Ohio State University should have read 38, not 18.

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Friction Tape Produces Glow Discharge

RECENTLY I found it necessary to repair an outdoor telephone wire after dark. In the course of rapidly peeling off a piece of ordinary electrician's "friction" tape, I noticed that a rather bright electrical discharge occurred along the line of contact of the roll and the straight strip during detachment. Teachers interested in simple demonstrations of electrostatic phenomena may find this useful. If the tape is rewound after the first unwinding, the phenomenon is much less intense, since the adhesion between strip and roll is now weaker. Various brands of tape may differ in their electrostatic behavior. I used the "Homart" brand sold by Sears, Roebuck and Company.

One might speculate that at the instant the thin filaments of rubbery adhesive sever, the resultant pairs of sharp points would have a high potential gradient from one to the other if there were any unbalance of electric charge between them. Finding a reason for a surplus of

charge on one surface seems a little difficult with this mechanism, since the material is presumably homogenous and symmetrical. Possibly the tension produces oriented electrostatic dipoles which leave a net charge after fracture. It is known, for instance, that soft iron fractured in tension is weakly magnetized because of orientation of magnetic dipoles. This hypothesis would not, however, provide a means for determining the *sense* of the field, and one would expect as many positive points as negative on one surface.

Asymmetry of materials can be introduced if it be assumed that the adhesive remains with the strip being removed, in which case separation occurs between cloth and adhesive. A definite sense of polarization now exists, and the production of free charges is no more mysterious than in the case of classical materials such as rubber and silk.

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ANNOUNCEMENTS AND NEWS

Book Reviews

Aspects of Form, A Symposium on Form in Nature and Art. Edited by LANCELOT LAW WHYTE. Pp. 249+ix, Figs. 77, 14.5×22 cm. Pellegrini & Cudahy, New York, 1951. Price $6.00.

According to the introduction this volume was initially conceived as a catalog to an exhibition on "Growth and Form" at the Institute of Contemporary Arts, London. It consists of essays by eleven specialists on topics which range from crystallography and astronomy to Gestalt psychology and visual art. L. L. Whyte, known perhaps to the readers of this Journal for his *Critique of Physics* and *Unitary Principle in Physics and Biology*, provides an introduction, a chronological survey, and a bibliography. Herbert Read supplies a short preface.

What can be said *for* the volume is that some of the constituent articles are usually informative and interesting and, in the opinion of at least one reader, are models of concise, technical exposition. The essay by F. G. Gregory on "Form in Plants," that by Hugh B. Cott on "Animal Form in Relation to Appearance," and that by Konrad Z. Lorenz on "The Role of Gestalt Perception in Animal and Human Behaviour" certainly fall within this category, and other readers might wish to include the essay by Joseph Needham on "Biochemical Aspects of Form and Growth" and that by W. Grey Walter on "Activity Patterns in the Human Brain" in this group. It can also be said in favor of this enterprise that it is an initial attempt to correlate the data and methods of science and art by a comparative study of a topic of fundamental importance in both disciplines.

Some of the contributors unfortunately have not considered it their responsibility to make an explicit correlation or comparison and have confined themselves to a study of form in the sense in which the word is used technically in their own specialized discipline. This is particularly true of the authors mentioned in the preceding paragraph. There is no evidence in Gregory's essay that he was aware that "form" could have any other meaning than that which it has in plant morphology. A similar statement might be made of S. P. F. Humphrey-Owen's essay on "Physical Principles underlying Inorganic Form." C. H. Waddington in "The Character of Biological Form" does venture on a comparison of organic form and artistic form, but his comparison eventuates merely in a distinction of the two with no attempt at a precise account of the exact differences. Rudolph Arnheim in "Gestalt Psychology and Artistic Form" attempts to demonstrate the relevance of Gestalt psychology to the explanation of our perception of artistic form, but I cannot say that I found his account very persuasive or thorough. The most successful essay in the volume is that of E. H. Gombrich, "Meditation on a Hobby Horse, or The Roots of Artistic Form," which is written with wit, gaiety, and erudition and an awareness of those aspects of linguistics, philosophy, and psychology which are relevant to the problem of the origin of form in primitive art.

The efforts of the editor to provide the requisite synthesis seem to me unsuccessful. The bibliography is useful, but the chronology is of very little value, if any. It contains the statements: "(Plato) discovers conic sections" and "125 (B.C.) Hipparchus proposes the use of geometry to describe the planetary motions," to mention no others. An adequate introduction to such a symposium should contain either (1) a precise account of the different senses in which the word "form" is used in these diverse disciplines, and this would in itself constitute a kind of correlation; or (2) some description of the causal or logical relationships between the forms of one discipline and those of at least one other; or (3) a discussion of form as a metaphysical concept of which mathematical form, logical form, atomic structure, organic form, Gestalten, and artistic form would be "concrete interpretations." This last is roughly Whitehead's method of discussing forms as "eternal objects." If this general criterion is accepted, Whyte's rather facile synopsis is inadequate.

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Interlingua-English, a Dictionary of the International Language. ALEXANDER GODE. Pp. 416+lxiv. Storm Publishers, New York, 1951. Price$ 5.00 (buckram) and $3.00 (paper, sold only outside the United States): and

Interlingua, a Grammar of the International Language. ALEXANDER GODE AND HUGH E. BLAIR. Pp. 118+x. Storm Publishers, New York, 1951. Price $3.50 (buckram) and $2.50 (paper).

The vocabulary of science and engineering forms the basis for this dictionary, for of the 27,000 words it contains 17,000 are scientific and technical terms. The remaining words are of the general vocabulary type and are words which already have wide international usage. The grammar is simpler than that of any national language and can be learned in a few days. An *English-Interlingua Dictionary* is in preparation.

The publication of these books represents a long step toward the goal of a common language for science. The present situation in which scientific results appear in many languages greatly retards the advancement of science and should logically be replaced by publication of scientific papers in a single language. For this purpose, it seems probable that a neutral auxiliary language would be more readily and universally adopted than would any one of the national languages. If the scientist had to learn only one language, in addition to his native tongue, he would be able to learn it well, whereas now he can never hope to become proficient enough in all the languages in which scientific results appear to read the papers adequately.

The basic principle in interlingua is that ordinary words already in the more important languages can by standardization form the vocabulary of an international language which can be learned easily because of its regularity and simplicity. It is a language growing naturally out of present international practice and is less artificial than previously proposed auxiliary languages. Interlingua represents the development of conclusions reached by a committee, under the chairmanship of the internationally known chemist, Frederick G. Cottrell, appointed by the International Research Council and including representatives of many scientific societies.

As a first step, authors might distribute prepublication interlingua abstracts of their papers to other workers in their field in various countries. Then, an international abstracting service could be developed, and interlingua abstracts could accompany the papers in the regular scientific journals. Eventually, for the greatest progress in science, all scientific papers should be published in the international language.

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The Conduction of Electricity Through Gases. K. G. EMELEUS. Third edition. Pp. 99+x, Figs. 37, 11×17 cm. John Wiley and Sons, Inc., New York, 1951. Price $1.50.

This monograph of the Methuen series is a reissue with only slight alteration of the second edition published in 1936. In the text Professor Emeleus is concerned mainly with collective phenomena associated with low pressure gas discharges, particularly the glow discharge. Here one can become acquainted with the glow discharge from its initiation (sparking potentials) on through the various regions of the operating discharge, cathode phenomena to anode glow.

For all its small size the book represents a considerable reservoir of organized empirical observations without at the same time burdening the reader with details of experimental technique. It leaves something to be desired in that it ignores, for the most part, the advances in basic understanding that have occurred since 1936. Thus the reader might be left with the impression that the field of gaseous electronics is a poor relation among the sophisticated branches of modern physics in the sense that it is a body of descriptive, qualitative knowledge rather than a quantitative science. To the extent that this impression constitutes a challenge which may inspire new workers to help tidy up this old and fascinating subject, no objection should be made, for many fundamental and challenging problems remain to be solved in this field.

Although not using the new advances, i.e., those made since 1936, in the body of the text to any appreciable extent, Professor Emeleus does recognize that important progress currently is being made. For he uses this fact as an argument, in the preface to the new edition, for undertaking no extensive revision of the last edition at the present time.

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Mechanics and Properties of Matter. R. C. BROWN. Pp. 276+ix, Figs. 259, 14.5×22 cm. Longmans, Green and Company, London, New York, and Toronto, 1951. Price $2.25.

The author in the preface states that this book, the first of four volumes, covers the work up to the standard required by the Higher School Certificate and Intermediate Examinations of English universities. I understand from a notice in the book that the other three volumes are to deal with heat, sound and light, and magnetism and electricity, respectively.

This makes the reviewer's work difficult in that some topics that are mentioned casually in this book such as the kinetic theory of gases in connection with diffusion of gases will no doubt be discussed in greater detail in subsequent volumes. The work covered in this textbook is approximately the grade of work given at the University of Toronto as part of the first-year's work in physics for general science students. However, we feel it would be advantageous to change and to attempt a more general course in the first year so as to leave more time in the second year for modern physics. If such a change is adopted a book of this type would not be suitable as a textbook. On general principles, I consider it unfortunate to keep up the traditional subdivisions in physics when there is so much overlapping between them in the modern

treatment of the subject. The author has made no attempt to introduce the mks units but uses the cgs and the fps units throughout the book.

Despite the limitations referred to, this textbook has many valuable features. The proofs are clear and logical, appear to be remarkably free from errors, and are difficult enough to hold the interest of the better students in the class. The author has avoided the formal use of calculus but its concepts are introduced in such topics as velocity and acceleration. By means of graphs the reader is introduced to the equation:

$$\text{distance} = \int_{t_1}^{t_2} \text{speed} \times dt.$$

Among other things I am pleased to see a detailed treatment of the chemical balance including a discussion of the factors affecting its sensitivity. The average student has only a very hazy idea about this part of the work. The sections on fluids in motion, surface tension, and viscosity are excellent in the range the book attempts to cover. By omitting detailed descriptions of applications of physics the author has crowded a lot of useful information into a small space. Another valuable feature is the large number of problems pertaining to all parts of the work. Many of these are questions from examination papers set by various English examining boards. This should be helpful to teachers who wish to compare the relative standards for their own work with that of similar courses in England.

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The Theory of Atomic Spectra. E. U. CONDON AND G. H. SHORTLEY. Pp. 441+xiv, Figs. 68. Cambridge University Press, London and New York. Reprinted with corrections, 1951. Price $11.00.

Everyone interested in atomic and nuclear spectroscopy is greatly indebted to the Cambridge University Press for reprinting *The Theory of Atomic Spectra* by E. U. Condon and G. H. Shortley. A genuine shortage of the book has developed, as shown by the worn condition of the few volumes available in certain laboratories. In fact the second-hand value had reached $15 before the recent reprinting. The great value of the book to the theoretical physicist is amply demonstrated by the many references to "TAS" in the literature. We can all join the authors when they regret that other duties have prevented their revising the first edition. This is one more sacrifice on the altar of a troubled world. However, practically all of the errors and misprints have been corrected, which will be a great help to people basing their calculations on the extensive tables.

The *Theory of Atomic Spectra* treats the energy levels and transition probabilities of complex atoms with and without electromagnetic fields. Wave mechanics and the matrix algebra of angular momentum operators are used rather than group theory. It has always seemed remarkable that by starting with the principles of electromagnetic theory one can calculate the energy levels of an atom with pencil and paper to almost the same precision with which they can be measured by exciting the atom in some sort of electrical discharge and photographing the spectrum with a diffraction grating.

It is interesting to think of some of the developments that might have been included in a revision. Probably the most spectacular advance has been the discovery by Lamb and Retherford of the errors in the Dirac theory of hydrogen. Certain energy levels which coincided according to the Dirac theory are found to have separations in the microwave region which brings in a whole new technique for measuring level differences. This discovery has given great impetus to the development of quantum electrodynamics with the aid of which Bethe, Schwinger, and others were able to calculate the observed separation of the levels. The theory also accounted for a slight increase in the gyromagnetic ratio of the spinning electron, which Kusch and Foley found to be larger than the value, demanded by the Dirac theory, of twice the gyromagnetic ratio for the orbital motion.

Secondly, in a brilliant series of papers, Racah has extended the theory of matrix algebra to that of tensor algebra. This development allows the prediction of the states of equivalent electrons with the aid of fractional percentage coefficients with the same ease as that of nonequivalent electrons. The spectra of *f*-electrons which are currently being analyzed can now be calculated without the formidable algebra of the matrix methods. Thirdly, the calculations of wave functions from the Hartree and Fock equations have been completed for many more atoms, and with the advent of electronic calculating machines will probably be extended rapidly in the next few years.

In the fourth place, considerable attention has been given to calculating hyperfine structure, since the information it gives about nuclear radii is of interest to the nuclear physicist. These calculations as well as others suffer from a lack of preciseness of the wave functions; and there have been several attempts to remove some of the approximations of the Schroedinger theory, such as, for example, the separation of variables. These efforts should prosper as the electronic calculators become more widely available. These developments do not replace the material in *The Theory of Atomic Spectra* but are based on it. Furthermore, any detailed study of nuclear energy levels on the basis of particles moving in an average central field is carried out by the methods of atomic spectra. The only essential difference between atomic and nuclear spectroscopy is that the nucleus has two types of particles which leads to the introduction of isotopic spin with a resultant increase in the array of energy levels. The tables of Condon and Shortley are becoming even more dog-eared. The field in the nucleus is also flatter, and the potential is not even known in principle.

It is clear that to understand the present developments one must read *The Theory of Atomic Spectra* to lay the foundation. It is not a book for the beginner, but should rather be preceded by an introductory book on quantum mechanics which will invariably say after treating the simple problems of atomic structure "for a more detailed discussion of this section and the next see E. U. Condon and G. H. Shortley, *The Theory of Atomic Spectra*."

C. W. UFFORD
University of Pennsylvania

Einführung in die theoretische Physik. Third volume, Second part, Quantum Theory. Second revised edition. CLEMENS SCHAEFER. Pp. 510+vii, Figs. 88. Walter de Gruyter and Company, Berlin, 1951. Price DM 40.

This book is an almost unchanged reprint of the first edition of 1938. It represents the concluding volume of the author's monumental four-volume "introduction" which, since the middle twenties, has been the standard encyclopedia of theoretical physics in Germany. The level of this present volume has been kept the same as that of the other volumes, namely, of the first- to second-year graduate student.

The amount of material covered is considerable: Chapters 1–3 deal with the quantum theory of phase integrals, Chapters 4 and 5 give a vector-model theory of atomic spectra, Chapters 6–9 contain the Schrödinger type of wave mechanics, and Chapter 10 treats the Dirac equation of the electron. The following topics are not discussed: collision phenomena, the W.K.B. method, the theory of metals, the quantum theory of radiation, and a treatment of particles of spin other than $\frac{1}{2}$. I should like to mention this in passing, merely to indicate how the range of topics is circumscribed.

The presentation shows a masterful clarity. As in the other volumes, the author has taken great pains not to intimidate the beginner with difficult mathematical notions or to discourage him by not explaining the intermediate steps of a calculation. The works of L. de Broglie seem to have influenced the presentation to a not inconsiderable degree. On the negative side, the reviewer is forced to note a general lack of original subject matter treated in an original manner, but, when dealing with the standard material of this volume, perhaps it is difficult to be original without exceeding the level of the book. Usually, at least exercises to be done by the reader give the author a chance to impart a personal stamp to a textbook; however, such problems are totally absent from this book. Nevertheless, I must confess that these are minor objections to an extremely well and competently written book. The German style employed is fortunately devoid of lengthy and involved sentences or difficult vocabulary. The book should therefore find favor with young graduate students whose "scientific German" is well known to be not too firm.

OTTO LAPORTE
University of Michigan

The Interpretation of X-Ray Diffraction Photographs. N. F. M. HENRY, H. LIPSON, AND W. A. WOOSTER. Pp. 258+ix, Figs. 223, 19×25.5 cm. D. Van Nostrand Company, Inc., New York, 1951. Price $8.50.

This book has been written for "students and research workers" as a guide to the theory and practice of the interpretation of diffraction patterns. Of the authors, N. F. M. Henry and W. A. Wooster are associated with the Department of Minerology and Petrology, Cambridge University, and H. Lipson with the College of Technology, Manchester University.

Two introductory chapters on the geometry of crystals and on the production and properties of x-rays are followed by a short chapter on the theory of x-ray diffraction. Here the Laue equations are developed before the Bragg law, which is the correct historical, but not pedagogical, order. The theory and use of the reciprocal lattice and sphere of reflection are well presented, but it is surprising to find that the name of Ewald is not even mentioned in this connection. For that matter, neither are the Laue equations credited to Laue.

Following this introductory matter, there are some nine chapters which constitute the real meat of the book. These describe, in considerable detail, the theory and practice of producing and interpreting rotation, oscillation, Weissenberg, Laue, and powder photographs as well as photographs made to determine the preferred orientation present in wire and sheet. This material is very clearly written and well illustrated by an abundance of drawings and practical examples fully worked out. A feature of this book is the reproduction of charts and x-ray photographs to the proper scale so that the student may use the former to solve the latter. The interpretation of powder photographs is exceptionally well discussed, particularly as regards analytical methods of indexing powder photographs of tetragonal, hexagonal, and orthorhombic crystals. These analytical methods are not generally enough known, and it is good to see them set down in some detail. Less satisfactory is the treatment of the Laue method in that too much attention is paid to symmetrical, or nearly symmetrical, Laue photographs and not enough to Laue photographs of crystals not having an axis parallel to the x-ray beam. The orientation of such crystals is most rapidly determined by the use of a back-reflection Laue photograph and a Greninger chart, but this method is not here described.

The treatment of diffracted intensities is not entirely satisfactory. On the experimental side, the photographic and ionization chamber methods of measurement are given, but no mention is made of Geiger counters. On the theoretical side, the calculation of diffracted intensities is not treated in a sufficiently fundamental manner. The important subject of diffuse reflections, resulting from thermal vibration or structural faults of one kind or another, is barely mentioned.

The authors of this book have not solved the main problem which the writing of any book presents, namely, what to put in and what to leave out. They have decided not to touch on the theory of space groups and yet space groups are mentioned, without definition, at various places in the text, and one example is even given of space group determination. This matter would be entirely unintelligible to any reader not acquainted with space group theory.

All in all, however, this book's good points outweigh its bad points. At least two-thirds of it can be highly recommended to students and research workers alike as a good text and reference book, but students beginning the subject will find some sections far from clear and informative.

B. D. CULLITY
University of Notre Dame

The Thermodynamics of the Steady State. K. G. DENBIGH. Pp. 103+vii, 11×17 cm. John Wiley and Sons, Inc., New York, 1951. Price $1.75.

The classical subject of thermodynamics, based in principle on the concept of the statistical ensemble, is severely limited in its applicability because it deals only with "quasi-static" processes. Real processes of interest are invariably irreversible, and of these classical thermodynamics is able to furnish only semiquantitative descriptions. In recent years, however, it has been increasingly appreciated that the statistical ensemble approach is not intrinsically limited to equilbrium situations, and that irreversible processes are, in fact, within the proper scope of an extended thermodynamics. Whereas the prediction of absolute rates of reactions must necessarily be based in kinetic considerations, there are a number of other aspects of a dissipative process which are essentially thermodynamic in nature.

The entree to the thermodynamics of irreversible processes is provided by the correspondence between macroscopic dissipative processes and the decay of spontaneous fluctuations in equilibrium ensembles. This relationship permits the conventional theory of equilibrium fluctuations to be reinterpreted as a thermodynamics of irreversible processes, and further developments of irreversibility theory are equivalent to extensions of fluctuation theory. Denbigh's monograph is primarily devoted to a discussion of the pioneer theorem in this field, which was developed in 1933 by Onsager, utilizing this correspondence with equilibrium fluctuations. The Onsager reciprocity theorem establishes a certain symmetry in the mutual interaction of two simultaneous irreversible processes, and provides a theoretical basis for the Kelvin relations among the thermoelectric coefficients, for the Soret effect in the thermal diffusion process, and for a variety of similar irreversible processes. As the intrinsic power of this fundamental theorem has been largely unappreciated for so long a time, it is a distinct pleasure to welcome this attempt by Denbigh to popularize it in the chemical literature.

The difficulties of summarizing the essential points of a rapidly expanding science in a small monograph are evident, and it is inevitable that certain aspects of the subject shall be under-stressed. Thus the very title *Thermodynamics of the Steady State* rejects a large area of irreversible thermodynamics. The steady state actually plays no unique or special role in the general subject, and the applications to thermal migration phenomena and diffusion potentials as presented in Denbigh's monograph, as well as the various other applications which have been made to date, deal with steady state processes only as a matter of convenience rather than of principle. Again, Denbigh has chosen to present the Onsager theorem without derivation, but he has devoted a very considerable portion of the book to a discussion of the basic principles upon which it rests. In this connection I would feel that the basic role of the correspondence with fluctuation theory could have been more explicitly acknowledged. Here also it is to be regretted that frequent reference is made to kinetic theory to provide justification for the thermodynamic arguments, whereas the interesting and significant theory of irreversible statistical mechanics is ignored.

Some of the most interesting aspects of the Onsager theorem appear only in the presence of a magnetic or Coriolis field. The ultimate basis of the reciprocity relations lies in the time symmetry of the quantum dynamical laws in the absence of magnetic or Coriolis fields, and the reciprocity is destroyed (in its simple form) if such fields are present. The omission of this aspect of the theory is perhaps the most regrettable concession to the limitations of space.

Some forty years before the work of Onsager, Lord Kelvin presented an argument suggesting the famous Kelvin relations among the thermoelectric effects. Although Kelvin was himself acutely aware of the lack of rigor of this argument, and stressed that it was presented only because of its suggestive value, the method for many years provided the only available analytic approach to irreversible processes. The technique of application of the method was not unique, and its application to a large class of problems suggested a great many results, some of which were false, but some of which were true and very valuable. The arbitrary and indefinite nature of the method has been demonstrated particularly clearly in the application to the thermomagnetic effects. A very large number of relations have been "derived" in the literature, but only one has survived the test of experiment. On the other hand, a number of additional relations, unobtainable by the Kelvin method, have been established by the Onsager theorem. Nevertheless, the unhappy method still persists in elementary thermodynamic textbooks, and engenders mystical metaphysical arguments as to what "portion" of a given process is irreversible and what portion is reversible. It is particularly regrettable that Denbigh has devoted two chapters, out of a total of six, to this method and to the demonstration of its "equivalence" to the Onsager theorem.

It is to be hoped that Denbigh's work will effectively introduce the subject of irreversible thermodynamics into the chemical literature and will herald a growing awareness of the powerful potentialities of this expanding field.

HERBERT B. CALLEN
University of Pennsylvania

Modern Magnetism. L. F. BATES, Third edition. Pp. 506+xii, Figs. 134. Cambridge University Press, Cambridge, England, and New York, 1951. Price $5.50.

The first edition of *Modern Magnetism* which appeared in 1939 was written with the firm conviction, acquired by the author during his many years of university teaching, that the fundamental experiments and techniques employed in magnetic investigations should be presented together with the theory in order to give a more thorough understanding of the subject. With this thought in mind practically the whole field of magnetism, starting with the early fundamentals and ending with the latest developments, was covered, with theory and experiments blended

together in a concise and lucid manner. Undoubtedly this method of presentation has contributed a good deal to the appeal of the book. In general, engineering applications were not dealt with, and magnetic materials were not covered very extensively.

In a second edition, in 1948, the author brought the book up-to-date by adding new material which had appeared in the intervening years. The third edition which has just been published is a more extensive revision which has included the important developments up to the time of publication. A considerable amount of new material has been added in two chapters—one dealing mainly with paramagnetic, nuclear, and ferromagnetic resonances and the other with magnetic domains and domain patterns which show the outline of the domains; the latter is a field in which Professor Bates and his students have been actively working and to which they have contributed substantially. It is quite fortunate that this subject, which is either entirely neglected or inadequately presented in most textbooks, has been treated by one who knows the subject thoroughly and is skilled in the art of presentation.

In this chapter, dealing with magnetic domains, the author explains why Weiss first introduced the concept of domains, and he discusses the various energies involved in domain structure and, in special cases, the size, shape, and geometrical arrangement that the domains assume in order to minimize the total energy involved. He also describes the magnetic powder pattern technique, originally due to Bitter and developed by others, for observing the domain structures. Special cases are discussed for which this technique provides a means of making quantitative measurements of domain size as a function of applied field. An attempt is made to correlate the changes in magnetization which occur while traversing the magnetization curve and hysteresis loop with domain boundary displacements and rotations in soft magnetic materials. Single domain particles, which are now of considerable practical interest in the manufacturing of permanent magnets are also discussed.

Modern Magnetism is an excellent book for students and a valuable aid for those doing research work in the field of magnetism.

H. J. WILLIAMS
Bell Telephone Laboratories

New Members of the Association

The following persons have been made members or junior members (*J*) of the American Association of Physics Teachers since the publication of the preceding list [Am. J. Phys. **20**, 288 (1952)].

Abraham, Marvin (*J*), 1102 Longfellow Ave., Bronx, New York, N. Y.

Arfken, George Brown, Jr., 203 Virginia Rd., Oak Ridge, Tenn.

Austermann, Richard Wagner, 724 N. Hilton St., Baltimore 29, Md.

Baumel, Philip (*J*), 1664 Macombs Rd., New York 53, N. Y.

Baldwin, Leonard Bush, Jr., 701-A Chestnut St., East Lansing, Mich.

Beauchamp, Edwin Knight (*J*), 691 Stevens Ave., Ely, Nev.

Bennett, Robert Bowen, 894 South 21st St., Salem, Ore.

Brieske, Phillip Richard (*J*), 775 North 23rd St., La Crosse, Wis.

Carr, Herman Y., Department of Physics, Harvard University, Cambridge 38, Mass.

Clarke, Helen Ayres, 450 So. 4th St., Maplewood, N. J.

Claypool, William Jacob (*J*), 643 Wood St., California, Pa.

Connelly, Donald B., 44 Schopman Dr., Schenectady, N. Y.

Council, Francis Elbert, Jr. (*J*), Box 793, Texas Christian University, Fort Worth 9, Texas

Dixon, Peggy Ann (*J*), 21B Lantz St., P.O. Box 558, Edgewood, Md.

Driscoll, John Francis (*J*), 152 E. Central St., Natick, Mass.

Dropkin, John J., 85 Livingston St., Brooklyn 2, N. Y.

Dunnett, John Phillip, 17148 McDougall St., Detroit 12, Mich.

Eccles, Samuel Franklin (*J*), 245 Lee Ave., Reno, Nev.

Edwards, Donald Anderson, Lincoln University, Jefferson City, Mo.

Eklund, John Robert (*J*), 627 N. Kensington Ave., La Grange Park, Ill.

Feldman, Lt. David W., 713 Great Falls St., Falls Church, Va.

Ferguson, David Norman (*J*), 49 Walford Way, Charlestown, Mass.

Fine, Samuel Joseph (*J*), USNMCS (Sp. Prob.) Panama City, Fla.

Fung, Lai-Wing, New Jersey College for Women, New Brunswick, N. J.

Gay, Joseph William (*J*), 43 Gordon St., Uniontown, Pa.

Geil, Philip Herbert (*J*), 2941 S. Wentworth Ave., Milwaukee 7, Wis.

Guthrie, George Drake (*J*), R.R. 2, Bloomingdale, Ind.

Hunt, Frederick V., 44 Beatrice Circle, Belmont 78, Mass.

Hultsch, Roland Arthur (*J*), 209 S. Grant Avenue, Crawfordsville, Ind.

Joachim, Hans S., 505 Ocean Ave., Brooklyn 26, N. Y.

Jones, Arthur Pierce, 11 Brookside Ave., Delmar, N. Y.

Jones, Charles Byron, General Delivery, St. Andrew, Fla.

Katz, Louis (*J*), 1367 Plimpton Ave., Bronx 52, N. Y.

Kennard, Sister M. Scholastica, C.S.C., St. Mary's College, Notre Dame, Ind.

Kuder, Bernard, 4405 Vesta Ave., Baltimore 7, Md.

Larsen, Ezra Ben, Tarkio College, Tarkio, Mo.

McGinnis, Gerald Alan (*J*), Rm. 16, N. Dorm., Beloit, Wis.

Martin, George Dewey (*J*), 2200 Loring Pl., Bronx 53, N. Y.
Morrison, Robert Eugene (*J*), 534 Pine St., Steelton, Pa.
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O'Connor, Brother C. Leonard, F.S.C., De LaSalle College, LaSalle Ave., N.E., Washington 18, D. C.
O'Donnell, Thomas Jefferson, 2617 Nicholson St., Hyattsville, Md.
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Rosen, Nathan, Physics Department, University of North Carolina, Chapel Hill, N. C.
Schafer, Bert J. (*J*), 420 Memorial Dr., Cambridge 39, Mass.
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Schilling, John J. (*J*), 513 Commonwealth Ave., New York 61, N. Y.
Scovil, Georgiana Winthrop (*J*), Physics Department, Bryn Mawr College, Bryn Mawr, Penna.
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Te Selle, Henry L. (*J*), Densmore Lodge No. 18, Beloit, Wis.
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Tillotson, James Howard (*J*), P.O. Box 6177, College Station, Texas
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AMERICAN JOURNAL OF PHYSICS VOLUME 20, NUMBER 6 SEPTEMBER, 1952

RECENT MEETINGS

Kentucky Section

The annual meeting of the Kentucky Section of the American Association of Physics Teachers was held on April 19, 1952 in the Physics Building of the University of Louisville. The meeting, held in conjunction with the Kentucky Education Association, was attended by 20 members and guests. DR. EARLAND RITCHIE, *Centre College*, presided. The program consisted of the following contributed papers.

A demonstration of the triple point for water. EARLAND RITCHIE, *Centre College*.—This demonstration is done before classes in general college physics which may be studying heat, and also for classes in courses on heat and thermodynamics. A set of triple point curves is placed before the class and referred to in interpreting the conditions that are seen during the demonstration. A good vacuum pump capable of producing at least a four-millimeter vacuum is attached to a bell-jar plate. An evaporating dish containing about an ounce of concentrated sulfuric acid is first placed on the plate. Over the dish is placed a wire, glass, or porcelain spider to support a watch crystal which contains a few tablespoons of tap water. A bell jar with its ground edge coated with vacuum wax is set over the dishes and onto the bell-jar plate. The sulfuric acid serves as drying agent to absorb the vapor released during the evaporation of the water. As the pressure is lowered the water soon boils thus causing the temperature to drop. At a temperature near zero centigrade and a pressure of four millimeters of mercury a skim of ice should sweep over the water surface. A sudden jar may be necessary to cause the crystallization to begin. Continued pumping causes boiling under the ice as more water freezes.

Velocity of sound in free air. WILLIAM FULLER AND ROY ELLIS, *Centre College*.—This demonstration of the velocity of sound in free air was originally developed by Professor H. C. Jensen of Lake Forest College, and demonstrated at the 1951 Colloquium of College Physicists. It was also summarized in the 1951–52 Winter Issue of *Cenco News Chats*. A signal from an audio-oscillator is fed to an ear phone placed at the focal point of a parabolic reflector and to one set of plates of an oscilloscope. A microphone is placed at the focal point of a second reflector located several wavelengths from the first. The signal picked up from this microphone is fed through an amplifier to the other plates of the oscilloscope, thus producing a 1:1 Lissajous figure. As the distance between the reflectors (and attached equipment) is changed by a distance of one wavelength, the Lissajous figure goes through a complete phase change of 360°. This measured distance and known frequency are used to calculate the velocity. This equipment may also be used to calibrate the frequencies of an audio-oscillator.

Graphical description of perfect optical systems. RICHARD HANAU, *University of Kentucky.*—When describing graphically the imagery of perfect optical systems in intermediate optics, the method[1] of projective geometry is useful. This graphical method not only gives meaning to the purely analytical approach using projective transformations, but has the advantage over the usual first-order, ray-drawing graphical method, of making no assumptions as to the kinds of optical surfaces, the laws of refraction and reflection, or the paths of light rays. If the optical system is composed of surfaces of revolution, the system is a one-dimensional collineation in any plane containing the axis. The collineation is unique if three distinct pairs of corresponding points are given. The two self-corresponding points of any such collineation are the axial Bravais points. If the collineation is perspective (coincident nodal points) the center of perspection is the nodal point, and the axis of perspection is the principal line. Any image point is found by the method of perspection and section. For nonperspective systems, two perspections are made using one of the nodal points and the infinite axial point as centers.

[1] For example, see Southall, Astrophys. J. **24**, 156 (1906); or *The Principles and Methods of Geometrical Optics* (The Macmillan Company, New York, 1913).

A proposed experimental physics course for seniors. R. S. CASWELL, *University of Kentucky.*—An experimental physics course is proposed in which senior physics students would work individually on small research problems and prepare reports in the style of the appropriate physics journal. This course would help the student make the transition from the "set-up" experiments of elementary laboratories to actual research work. The experimental course would have certain advantages over the requirement of a bachelor's thesis: (1) The student's graduation need not be held up by the requirement of successful conclusion of the thesis, (2) fewer new problems each year would be required since experiments might be repeated at intervals. Classical experiments as well as modern could be performed. This course would give the student an opportunity to demonstrate initiative and ability for experimental research (as distinct from classroom work). The course might also serve as a place to develop experiments for other undergraduate laboratories.

The place of general education courses in the subject major. EARLAND RITCHIE, *Centre College.*—The college curriculum committee is now facing a question which was presented by the faculty. The portion of the problem which pertained to physics was the possibility of a part of the physical science course being substituted for one semester of general college physics. Under our present setup for the physical science course, at least one semester of time is given to physical topics which are approximate duplicates of many topics in the regular physics course. Many class demonstrations are used in common. The physical science course as it is now planned does not carry a laboratory. We are therefore suggesting revision of content and arrangement of subject matter in both the physical science and physics courses as follows: (1) Add laboratory to the physical science course. (2) Arrange second semester of physics to cover topics not touched on in physical science. (3) Accept four semester hours of credit in physical science as a substitute for first-semester general physics.

Construction of a recording comparator microphotometer. R. J. REITHEL, B. D. KERN, AND RICHARD HANAU, *University of Kentucky.*—A recording comparator microphotometer has been constructed for the purpose of measuring the density and position of lines on photographs of oscilloscope patterns resulting from scintillation spectroscopic detection gamma-rays. Components are the light source, plate carriage, and optical system of a Hilger microphotometer, a Celectray recording potentiometer, an RCA 931A photomulitplier tube, and an Atomic Instrument Company, 2000-volt power supply. The Hilger instrument was modified so that it is immediately available for hand operation. A tube attached to the rear of the fine-drive shaft is engaged by the shaft of a synchronous motor which drives the carriage 0.0075 cm per minute. A microswitch attached to the motor mount opens all circuits when touched by the carriage. A 120-degree totally reflecting prism deflects the beam of the microphotometer through an auxiliary slit system. The output of the photomultiplier tube behind the slit activates the recording potentiometer. The record as printed by this instrument is a graph of position on the photograph, which is porportional to gamma-ray energy, against transmitted light intensity which is related to the number of counts.

The officers elected for 1952-53 are *President*, PAUL C. OVERSTREET, *Morehead State College; Vice-President*, CARL ADAMS, *University of Louisville; Representative to the Executive Committee*, AAPT, RALPH A. LORING, *University of Louisville; Secretary-Treasurer*, RICHARD HANAU, *University of Kentucky.*

The meeting was concluded with a discussion, during luncheon, of certain phases of the general education program.

RICHARD HANAU, *Secretary*

Southern California Section

The annual spring meeting of the Southern California Section of the American Association of Physics Teachers was held at the University of Southern California on Saturday, April 19, 1952. About forty members and friends enjoyed the day's program. It began with an invited paper on **Antennas** by DR. LESTER VAN ATTA of the *Hughes Aircraft Company.* The remainder of the morning was devoted to a visit to the First Annual Southern California Science Fair held in the Los Angeles County Museum in Exposition Park. The Science Fair, a contest of scientific exhibits prepared by high school students, was under the auspices of the Los Angeles County Museum. It was conducted by the California Science Teachers, Southern Section, and the Southern California Section of the American Association of Physics Teachers in cooperation with the *Los Angeles Times* and various colleges and scien-

tific organizations of Southern California. There were a total of 239 separate exhibits covering all fields of science and its application to modern life.

After luncheon the following ten-minute contributed papers were presented:

Conditions underlying the control and utilization of energy. WALTER D. O'CONNELL, *University of Southern California.*—Systems in metastable equilibrium constitute potential sources of energy. This energy remains stored so long as the displacement of the system is less than a certain critical value at which the equilibrium shifts from stable to unstable. "Displacement" here refers to change in the pertinent variable (position, temperature, pressure, voltage, charge, mass, number, etc.). The metastable threshold is generally crossed with the aid of concentration mechanisms (lever, pin, hammer, lens, etc.), which produce localized high intensity effects. If the transformation thus initiated goes to completion uncontrollably (explosion), the concentration mechanism may be called a trigger. If the rate of transformation is limited by velocity-dependent resistance, the initiating mechanism is a switch (light switch, bath tub stopper, etc.). A throttle represents a still more detailed control of the flow, which may be governed by a cybernetic negative-feedback element (governor). Each of these modes of release may be subjected to various degrees of control. For example, an automobile utilizes an explosive release by opposing it with a piston constrained in its motion by a cylinder. Some utilization of an already existing flow can be achieved by resistance alone (a log floating down a stream, a leaf in the wind). But the controlled utilization of a flow requires the addition of at least one constraint. Thus the sailboat differs from the log or leaf: The opposition that its sail offers to the wind produces motion only in the direction selected by the keel, whose action depends upon the presence of the additional phase, water. Here the sail represents the opposing resistance which initiates the transformation, while the keel is the constraint directing its utilization. In summary, the controlled utilization of energy seems to exhibit the following pattern (or some sections of it): A metastable equilibrium is lifted to its threshold by a concentration mechanism, a controlled release is maintained by some feedback device, and the resulting flow is opposed by a resistance guided by a constraint.

Trends in survey physics in the East, good and bad. WILLARD GEER, *University of Southern California.*—A new way of teaching courses in physics for liberal arts students who will take no further work in science has appeared in the East and is exerting strong influence among many universities. This case-study method of teaching as advocated by some at Harvard University generally consists of a critical study of the thinking of a scientist at important turning points in the history of physics. Newton, Kepler, and Copernicus formulating their theories, or Boyle writing his thesis on the "Spring and Weight of the Air" are often chosen. Experiments are generally connected with the topic chosen, such as dropping unequal weights, as Newton did. A critical review and discussion of all the influencing factors of that time gives the student the best chance to develop critical thinking—such is the argument for this type of course.

Opponents of the case-study method do not say it cannot teach critical thinking, but say it is wasteful and inefficient and that historical philosophy or logic is not a substitute for a knowledge of the fundamental laws of physics surrounded by the applications of which modern man must, willy-nilly, live. Unless the cases studied are expanded into historical research the classes are not kept busy, and this lost time, the opponents say, could be better spent in learning useful information, even in the most "smorgasbord" type of course.

This speaker feels that critical thinking can be taught by the "block-and-gap" method as well as with the "case study" type of course. Student polls at the University of Southern California show that they want this course to provide useful information for the present and future and that they have little desire for understanding the troubles of the early scientists.

A project approach for the general physics laboratory. BURTON HENKE, *Pomona College* (Introduced by Charles A. Fowler).—With the purpose of encouraging student initiative in the laboratory as well as presenting the experimental problem in a more realistic manner, Pomona College has inaugurated a project approach in the general physics laboratory. The same experiments that were used in the conventional laboratory are combined to constitute as nearly as possible miniature research projects, with no printed instructions other than a statement of the problem and a list of available equipment that might be used for its solution. The role of the instructor in this kind of laboratory was discussed in detail.

Can we teach physics? WILLIAM P. BOYNTON, *Whittier, California.*—Teachers of physics have in the past fifty or sixty years seen a phenomenal growth of the content of their subject, and have faced the necessity of modifying their techniques. Social and economic progress has expanded student enrollment from a limited list of young people eager for an education, or at least cheerfully cooperative, to universally required high school attendance with a high percentage of continuance through junior college. Under these conditions our best efforts to present our subject seem to accomplish merely a reasonably thorough exposure to our subject. The ultimate results depend largely upon the susceptibility, interest, and industry of the student.

Illusional motion. LAURENCE E. DODD, *University of California at Los Angeles.*—Based on studies of the stroboscopic effect made by the writer some years ago, the present paper discusses particularly stroboscopic motion with direction variable relative to that of the stroboscopic screen.

Some remarks concerning the Chicago meeting and the Buck Hill Falls conference. VERNON L. BOLLMAN, *Occidental College.*

A dynamic ray tracer for thin lenses and spherical mirrors. HENRY A. KNOLL, *Los Angeles College of Optometry* (Introduced by Arthur W. Nye).—When tracing rays for thin lenses and spherical mirrors a ray parallel to the optic axis will, after refraction or reflection, pass, really or virtually, through the secondary focal point. A single ray can be used for all real and virtual objects. A second ray passing through the optical center of a thin lens, or through the center of curvature in the case of a curved mirror, may be used to determine the position and size of the image. This ray is referred to as the center ray.

On the ray tracer described, the parallel ray is drawn to scale for each of the four systems: positive and negative lenses, and positive and negative spherical mirrors. The lenses are shown on one side of the ray tracer, the mirrors on the other. The center ray is represented by a line scribed on a thin piece of plastic which rotates about the optical centers of the lenses and the centers of curvature of the mirrors. In tracing rays to find the image, the center ray is simply rotated to intersect the parallel ray at the desired object position and the intersection of the center ray with the reflected or refracted ray gives the position and size of the resulting image.

The ray tracer has a grid background which permits measurement of the object and image distances in tenths of the focal length. The image size and the magnification can also be computed quickly and accurately. The dynamic relationship between object and image can be shown very clearly by rotating the center ray.

Comparative physics. WALTER D. O'CONNELL, *University of Southern California.*—Underlying any physical theory will be found ideas that are essentially qualitative in nature. Physics at any given time may be regarded as a quantitative structure built out from such root ideas. The formulation and testing of these broad qualitative principles is thus an integral part of the scientific process. The science teacher who wishes to offer his students some first-hand contact with the formulative aspects of physics may profitably consider several comparative methods. (1) A student can be encouraged to collect lists of phenomena that illustrate in different ways some idea that he personally wishes to investigate. For instance, if he is interested in antigravity and flying saucers, he may compare already known methods of opposing the force of gravity. A few items in his list might include: a table, a man's arm, a blimp, a helicopter, a kite, an electromagnet, the moon, etc. These are then examined individually for energy expenditure, visible means of support, etc. (2) Parallelisms—comparisons of the forms of laws governing apparently unrelated phenomena—may be built up by the student, compiling tables of similar and contrasting facts concerning two or more topics. When he first approaches electrostatics and magnetostatics, for example, quite an extended table can be constructed, which makes the facts of gravitation all the more interesting through their contrast. Here antigravity seems neither predicted nor denied explicitly. (3) As parallelisms are extended far enough, actual physical interconnections are often discovered. Oersted's discovery appears as the first break in the wall separating electricity and magnetism. Faraday's search for electromagnetic induction emerges as an attempt to discover a symmetry in this interaction. His original guess, his actual discovery, as well as Maxwell's additions and predictions, can all be described by sentences involving exact verbal interchanges. It is as if Faraday had supposed that one can interchange the brackets in: "A steady (electric current) produces a steady (magnetic field)"; while his discovery might have been accurately predicted by interchanging the brackets in: "A steadily moving (electric charge) exerts a force on a fixed (magnet)." After such examples the student is asked to find statements summarizing Faraday's observations which when interchanged will yield Maxwell's assumptions. Those interested are asked to keep notebooks of scratch work developing some original idea by applying such methods. Thus, for example, a first-year student can in his own way begin the search for a unified field theory.

At the ensuing business meeting of the section it was voted to continue to sponsor the Science Fair; and the following officers were elected for the year 1952–53: *President*, CHARLES A. FOWLER, JR., *Pomona College; Vice-President for Colleges*, WALTER O'CONNELL, *University of Southern California; Vice-President for Junior Colleges*, LESTER M. HIRSCH, *East Los Angeles Junior College; Vice-President for High Schools*, RAY B. POTTER, *George Washington High School, Los Angeles;* and *Secretary-Treasurer*, DAVID F. BENDER, *Whittier College.* As usual, an executive committee meeting concluded the activities of the day. The 1952 fall meeting is scheduled for Saturday, October 4, at the California Institute of Technology.

DAVID F. BENDER
Secretary-Treasurer

Chesapeake Section

The annual spring meeting of the Chesapeake Section of the American Association of Physics Teachers was held at Catholic University of America, Washington, D. C., on Saturday, April 5, 1952. About sixty members and friends from Delaware, Pennsylvania, Maryland, District of Columbia, and Virginia were present.

The new officers for 1952–53 are: *President*, R. R. MEIJER, *George Washington University; Secretary-Treasurer*, J. R. HEVERLY, *U. S. Naval Academy; Member of the National Executive Committee*, G. E. C. KAUFFMAN, *University of Delaware; Members of the Executive Committee*, F. L. TALBOTT, *Catholic University*, R. D. MYERS, *University of Maryland.*

The program consisted of three invited papers and eleven contributed papers.

Invited Papers

Measuring with the electron. L. MARTON, *National Bureau of Standards.*

Solar eclipse movies at Khartoum. D. J. LOVELL, *Naval Research Laboratory.*

Progress in better living. L. P. SHANNON, *E. I. du Pont de Nemours Company.*

Contributed Papers

On the statement of definitions and laws in electricity and magnetism. R. R. MEIJER, *George Washington University.*—It is difficult to state some of the laws or definitions of the fundamental concepts in electricity and magnetism. Examples are emf, flux density, Lenz's law, and Ohm's law. The problem is to be concise and at the same time precise in addition to imparting physical meaning to each statement. Solutions were suggested in some cases and questions raised about other cases.

Records and recommendations. HALSON V. EAGLESON, *Howard University.*—Physics teachers frequently have the difficult task of recommending students for employment or for entrance in graduate or professional school. Selections must often be made from several students who have identical grades. There may be an attempt to base decisions on certain intangibles such as leadership, personality, and ability to follow directions. Attention is called to three types of record forms that have been of value in furnishing a picture of these intangibles that may be kept for reference when needed.

The teaching of related physics to the machine shop students. JAMES A. PARKER, *Armstrong Technical High School.*

Forensic expertness in science. WILMER SOUDER, *National Bureau of Standards.*

The mass spectroscopy laboratory at Howard University. HERMANN BRANSON AND W. N. MCINTOSH, *Howard University.*—The mass spectroscopy laboratory with a 60° Nier-type mass spectrometer constructed with the assistance of first-year graduate students is used to give realistic training on an active research project. The students study, in an interrelated manner, such topics as high vacuum technique (pumps, speed, gauges, leak testing), high and low voltage power supplies (regulation, shielding, insulation), electron emission, magnetic fields (gaussmeters, Hall effect), dc amplifiers, and ion optics. The spectrometer is now being used, for example, to measure the appearance potential of C^+ in a series of substituted methanes. Our data support the value of 5.9 ev for the heat of sublimation of carbon from graphite to the ^3P state. (Supported by ONR, N8-ONR-538).

Mathematical preparation and general college physics. J. H. TAYLOR, *Morgan State College.*—The ability of students to understand and carry out fundamental mathematical operations which arise in general college physics have been studied during the school year 1950–51 at Morgan State College. Although the sampling is very small, we have been able to predict rather accurately success or failure in general physics from the results of an examination prepared by this department. It is astonishing to find so many students exhibiting such weak understanding of mathematical fundamentals who have taken the necessary prerequisites to college physics. The results of this study reveal that about 80 percent of our failures can be traced to poor mathematical preparation.

Working up in a swing from rest. G. S. FULCHER, *Washington, D. C.*—Since working up in a swing is a familiar example of the use of muscular force to increase energy, students may be interested in an explanation of why the motions used by children are effective. Simple experiments at a playground showed that a child standing erect in a swing at rest can displace the center of gravity of the system and thus start swinging by pulling his body forward between the ropes and then waiting for the system to move back through the position of rest. This result can be explained by showing that the child's body can be replaced by a mass M at his center of percussion connected to a mass m at his feet by a rigid rod, and by showing that the ropes exert a force on m which is less than that exerted in the opposite direction on M by the muscles in the child's arms in the inverse ratio of the lever arms of the forces about the axis of support of the swing, just as though the ropes were rigid levers. After the swinging motion has been started, the amplitude can be increased either (1) by a series of properly timed tangential pulls and pushes exerted by the arms while the child holds his body rigidly erect or (2), if he is standing, by the child periodically raising and lowering his center of gravity with reference to the seat. Most children use a combination of these two methods.

A variation of the index of refraction experiment. G. M. KOEHL, *George Washington University.*—The paths of rays of light as they pass from air into a transparent medium are determined by some ray-tracing device such as the pin-alignment method or passing light from a straight-filament show-case lamp through a narrow slit which is parallel to the filament. The angles of incidence and refraction are measured. The sines of the angles of incidence are plotted as ordinates and the corresponding angles of refraction are plotted as abscissas. The slope of the resulting straight line is the index of refraction of the transparent material. The plot, which passes through the origin, shows that there is no refraction at zero angle of incidence. The critical angle of the transparent material is easily obtained from the plot.

Huygen's principle in the study of a sound field near a source. J. C. HUBBARD, *Catholic University.*

Standing ultrasonic waves in the calibration of screws. J. C. HUBBARD, A. J. GABRYSH AND HAL WATERS, *Catholic University.*

Laboratory experiment in measurement of capacitance. J. H. REAVES, *American University.*—A method of measuring capacitance, believed to be novel, is described. The

method makes use of an automatic mechanical or electronic switch, which alternately causes the charging and discharging of theu nknown capacitance. The resulting pulses of current are integrated by a galvanometer, and the galvanometer deflection is used to indicate the equality between the average value of the current pulses and the steady dc current from the same voltage source and a known resistance. This gives an absolute measurement of the capacitance in terms of the resistance and the switching frequency. The simplicity of the method makes it readily usable as a student laboratory experiment.

J. HOWARD MCMILLEN, *Secretary*
Naval Ordnance Laboratory

Southeastern Section, American Physical Society

The eighteenth annual meeting of the Southeastern Section of the American Physical Society was held at North Carolina State College, Raleigh, N. C. on April 10, 11, and 12, 1952. The registered attendance was 292. A program of eigthy-eight contributed and four invited papers was admirably arranged by a committee of which WALTER GORDY of *Duke University* was chairman. The abstracts of nine of the contributed papers, concerned primarily with the teaching of physics, appear below; the others will be published in an early issue of the *Physical Review*. At the annual dinner, served to 159 in the College dining hall, DR. G. B. PEGRAM, *Columbia University*, spoke on **Names and Personalities.**

The other invited papers were:

The North Carolina State reactor. C. K. BECK, *North Carolina State College.*

Nuclear quadrupole resonance in solids. RALPH LIVINGSTON, *Oak Ridge National Laboratory.*

New results on positronium. MARTIN DEUTSCH, *Massachusetts Institute of Technology.*

The section has chosen WALTER GORDY, *Duke University*, as its chairman for 1952–53. Other officers elected were J. H. COULLIETTE, *University of Chattanooga, Vice-Chairman;* DIXON CALLIHAN, *Oak Ridge National Laboratory, Secretary;* R. T. LAGEMANN, *Vanderbilt University, Treasurer.* New members of the Executive Committee are J. W. BEAMS, *University of Virginia*, and R. C. KEEN, *Louisiana State University.* W. G. POLLARD is the retiring Chairman.

DIXON CALLIHAN, *Secretary*

1. The production of professional physicists. MARSH W. WHITE, *The Pennsylvania State College.*—Data were presented to show the numbers of physicists graduating at the bachelor's, master's, and doctorate levels from 500 American colleges and universities during the last six years. Following World War II there was a tremendous increase in physicists-in-training at all levels. The numbers of bachelor's degrees in physics which were awarded reached a maximum of 3770 in the academic year 1949–50. For 1950–51 this number declined to 3250. There was also a sharp drop in the number of master's degrees, from 1065 in 1949–50 to 860 in 1950–51. At the doctorate level the production is continuing to rise, with 450 degrees in 1950–51, as compared with the all-time high of 402 in 1949–50. Total enrollments in the physics major are decreasing at all levels. Some significant data are available concerning academic scholarship in the colleges and universities of the Southeast, in comparison with other sections of the country.

2. Cgs or mks units for elementary physics. VINCENT E. PARKER, *Louisiana State University.*—The pedagogical advantages and disadvantages of the cgs and mks systems as used to teach elementary physics to beginning students are considered. Particular attention is given to the best method of teaching electrostatics, current electricity, and magnetism. The attempt of some recent textbooks in general physics to use both systems in electrostatics is especially unsatisfactory, eliminating the advantages of each system and compounding the difficulties of both. It is not apparent that the mks system is completely satisfactory in all branches of physics, or that it will eliminate the development of new systems of units especially suited to specialized fields of study. The author concludes that the cgs system is preferrable in the teaching of beginners.

3. Teaching pulley systems. ANGUS G. PEARSON, *Emory University.*—The analysis of pulley systems by counting supporting ropes directs the student's attention away from basic principles and leads to superficiality. The analysis of pulley systems by consideration of the forces involved is more instructive. It also permits the student to discriminate between proposed pulley systems which are workable, and those which are not. In order to direct the student's attention to basic principles it is advisable to present pulley systems which cannot be treated by "rope-counting." The Spanish Burton is a well-known example of such systems; several additional systems were presented.

4. Alpha-range experiments in the intermediate laboratory. RALPH A. LORING,* *Unversity of Louisville.*—The availability and usefulness of four types of alpha-range apparatus were discussed. Details were given of an ionization chamber-electronic electrometer set-up that has been built and used at the University of Louisville. The ionization chamber was built around a coffee can. Graphs taken from student reports were shown.

* Now research participant at Oak Ridge National Laboratory.

5. Development of a course in nuclear instrumentation. R. J. STAUVERMAN, *De Forest's Training, Inc.*—Many graduates of De Forest's Training, Inc., a technical school teaching electronics, have become employed in nuclear industries or related fields, such as instrument manufacture. Therefore, a need was apparent to offer a course dealing with instruments and their use, to prepare techni-

cians to enter these new industries. The course is directed toward study of electronic instruments used in nuclear work, their circuits, operation, capabilities, limitations, etc. These are studied in class, together with sufficient physics to acquaint the student with the field he is entering. The laboratory is equipped with instruments of many different makes and types, such as count-rate meters, scalers, particle detectors, etc. When possible the students make "bread board" models of representative circuits. Planning the course presented a problem, since there was no precedent to follow, no textbook, and little advice available. Most available material concerned results, rather then practices of instrumentation. After consulting textbooks and other pertinent sources, lessons were prepared by the school's staff writers, and laboratory material by instructors. At present the course is just beginning, and will be altered as necessary to fit the needs of students and the nuclear industries.

6. The education and training of health physicists. E. E. ANDERSON, *Oak Ridge National Laboratory.*—The advent of atomic energy and the continued expansion of the atomic energy program has given rise to an increasing need for trained health physicists. In the AEC Radiological Physics Fellowship Program the student spends an academic year either at Vanderbilt University or the University of Rochester, followed by three months of field work and research activities either at Oak Ridge National Laboratory or the Brookhaven National Laboratory. This program as well as the other training and educational activities of the Health Physics Division at ORNL helps to provide specialized graduate training to physicists and furnish competent health physicists to satisfy the increasing needs in this field.

7. The experimental reactor physics course of the Oak Ridge School of Reactor Technology. ERNEST D. KLEMA, *Oak Ridge National Laboratory.*—The list of 25 experiments performed this year by the students of the Oak Ridge School of Reactor Technology were presented and discussed briefly. The first group of experiments is mainly concerned with nuclear physics, while the latter part of the course is devoted to experiments for the measurement of quantities of importance in reactor design.

8. A student's neutron spectrometer. W. C. KOEHLER AND C. C. HARRIS, *Oak Ridge National Laboratory.*—A simple neutron spectrometer has been constructed for the use of students in the Oak Ridge School of Reactor Technology. An approximately monoenergetic beam of neutrons is selected by crystal diffraction from the continuous distribution of pile neutrons, with which a set of boron absorbers is calibrated. The BF_3 counter is set by optical methods at a known angle to the direct beam, and different neutron energies are selected by scattering from crystal planes of different spacing. Three convenient energies are selected from a small Cu crystal, so cut that the planes (111), (200), and (220) are successfully brought into position for reflection by rotation about a horizontal axis. A fourth energy is obtained from the 200 planes of rock salt. The wavelength covered is 1.4A to 0.64A, and this can be extended by proper selection of monochromators.

9. The scattering and absorption of neutrons. R. J. STEPHENSON AND C. C. HARRIS, *Oak Ridge National Laboratory.*—A Po-Be source emitting about 10^7 neutrons per second is placed at the center of a paraffin cube about 2 feet on a side. Aluminum tubes going near to the center provide ports for four neutron beams. Measurements are made on the back scattering of neutrons by several paraffin blocks using a BF_3 proportional counter as detector. A long BF_3 proportional counter placed in line with the neutron beam is used to measure the total cross section of various substances. The relative scattering cross section of various substances is determined at the Oak Ridge graphite reactor using a narrow beam of thermal neutrons and a BF_3 counter.

Oregon Section

The 59th meeting of the Oregon Section of the AAPT was held at Willamette University, Salem, Oregon, on March 1, 1952. Both morning and afternoon sessions were scheduled, the latter following a business meeting called to order by DR. WILLIAM L. PARKER, President of the Section.

It was resolved on motion by Dr. Varner that the next meeting be set tentatively for April 26, 1952, at the University of Oregon.

In response to inquiry concerning the desirability of making available tape recordings of papers presented at national meetings, it was the consensus that such recordings should be made on an experimental basis but only of papers not otherwise published.

The program included the following papers:

1. Report on the APS meeting, January–February, 1952. RAYMOND T. ELLICKSON, *University of Oregon.*

2. Report on the AAPT Council meeting, February, 1952. WILL V. NORRIS, *University of Oregon.*

3. Trends in general educational courses in physics. WILLIAM L. PARKER, *Reed College.*

4. Table of important events in the life of Michael Faraday. C. L. CHURCH, *Oregon State College.*

5. Teaching aids in first-year college physics. R. L. LINCOLN, *Oregon State College.*—Examples were given of classroom demonstrations of simple attention-getting experiments. These were designed to use simple equipment so that they could easily be performed afterwards by the student himself. The purpose of the demonstrations was to stimulate discussion of fundamental physical principles among students. The following subjects were included,—torque, Newton's third law, energy transformation, conservation of angular momentum, friction, equilibrium, surface tension, air pressure, P-V relation of Bernoulli's theorem, resonant air column, heat convection, "bending" light, vortex rings.

6. The construction of a diffraction grating spectrograph. R. L. PURBRICK, *Willamette University.*—The construction of a Wadsworth-mounted, diffraction-grating spectrograph employing a 21′ 10″ radius of curvature grating was described. The mechanical operation of the instrument and its optical characteristics were discussed. The instrument covers a range from 9000Å to 1800A in the first order with a linear dispersion of 5A/mm. Colored slides taken at various stages of construction were shown.

Two motion pictures of the McGraw-Hill Book Company series, "The Otto Cycle" and "Uniform Circular Motion" were shown.

FRED W. DECKER, *Secretary*

American Physical Society

New England Section

The spring meeting of the New England Section of the American Physical Society was held at Tufts College, Medford, Massachusetts on Saturday, May 24. The program consisted of four invited papers and fourteen contributed ten-minute papers. Abstracts of those of the latter group which have some interest for physics teachers are printed below.

Invited Papers

A few engaging aspects of current work in optics. EDWIN H. LAND, *Polaroid Corporation.*

Remarks on teaching of modern physics. F. L. FRIEDMAN, *Massachusetts Institute of Technology.*

Microwave properties of ferrites. C. L. HOGAN, *Bell Telephone Laboratories.*

Field emission microscopy. ERWIN MÜLLER, *Pennsylvania State College.*

Contributed Papers

1. A probe technique for measuring ion densities in the afterglow of a pulsed discharge. MORTON A. LEVINE AND WINSTON H. BOSTICK, *Tufts College.*

2. Direct observation of the presence of magneto-hydrodynamic waves in ionized He gas. WINSTON H. BOSTICK AND MORTON A. LEVINE, *Tufts College.*

3. Thermal conductivity of sapphire and rutile as a function of temperature. KATHRYN A. MCCARTHY, STANLEY S. BALLARD, AND EDWARD C. DOERNER, *Tufts College.*

4. Inelastic deformation (cold flow) of certain crystalline materials under flexural stress. LEWIS S. COMBES, STANLEY S. BALLARD, AND DAVID L. HONKONEN, *Tufts College.*

5. Effective mass of electrons in polar crystals. E. P. GROSS, *Massachusetts Institute of Technology.*

6. Measurements of the lateral distribution of extensive air showers. MORTON A. LEVINE, *Tufts College.*

7. Some demonstrations with a Projection thermometer. W. T. SCOTT, *Smith College,* and J. J. G. MCCUE, *Massachusetts Institute of Technology.*—Demonstrations of thermometry and calorimetry are essential to the teaching of temperature and heat concepts to elementary students. While the thermocouple is often useful, the electrical ideas involved complicate matters for most students, and the use of a direct-reading projection thermometer seems desirable. We have obtained a Westondial-type thermometer with a transparent dial (special order: cost 30 dollars) and have arranged a convenient projection system. The stem of the thermometer extends through a hole in a small mirror and is available for measuring the temperature of various 250-ml beakers of water. An arc light is used to project a clear image approximately 3 ft in diameter so that a large class can all read the temperature to 0.5 degree or better.

Simple demonstrations that illustrate the development of a temperature scale, the idea of the calorie, and the concept of specific heat will be discussed. The apparatus will be shown. Some consideration will be given to the way in which this material is used in the two general education courses in physical science now offered at Smith College.

8. An improved laboratory experiment on vacuum tubes. PHILIP J. BRAY AND NATHANIEL P. CARLETON, *Harvard University.*—The apparatus we have previously used for elementary laboratory instruction on the use of vacuum tubes had the circuit components mounted on a board with the actual wiring and smaller elements out of sight. This arrangement protected the student from the plate voltage and permitted a clear circuit representation to be drawn on the surface of the board. We feel, however, that such a scheme removes much of the experimental aspect of the laboratory work. In order that the student may see all components and wiring, we have mounted the elements of three basic units (tube, grid bias control with voltmeter, plate circuit with milliammeter) on separate plexiglass boxes. The student can follow the actual wires to each component and can connect the units themselves together with banana plug leads. He obtains by experiment the plate current *vs* grid bias characteristic for the tube and uses it to predict how the tube will amplify and rectify. He then introduces an ac signal and observes the output on an oscilloscope. One further Plexiglas unit, an RC coupling network, is used with two triode units, tuner, and loudspeaker to form a two-stage radio.

9. Two experiments for elementary physics laboratory: (a) conservation of energy, (b) decay of thorium emanation. J. W. STEWART AND R. A. ROSS, *Harvard University.*—In the first of these experiments, the law of conservation of energy is investigated by means of a car which is allowed to roll down a steel track. The time of descent for the car, the height and length of the inclined plane, the force of rolling friction acting on the car, and the mass of the car are measured by the student. The loss of potential energy, the final kinetic energy of the car, and the work done against friction are then calculated. The prediction based on the law of conservation of energy is verified within an experimental error of less than 5 percent.

The second experiment concerns the half-life for the α-decay of thoron and represents a modification of a long-established method. Thoron, produced continuously from

thorium oxide, is introduced into an emanation electroscope isolated from the source of the thoron. Successive initial rates of collapse of the leaves, each time recharged to the same potential, are then observed with telescope and stop-watch. The half-life can be obtained directly from a plot of the data.

10. The limitations of the experimental processes. EUGENE W. PIKE, *Raytheon Manufacturing Co., Waltham, Massachusetts.*—The experimental process, keystone of the discipline of physics, becomes itself the proper object of study to physicists to whom the war brought experience with operation analysis. One approach to this study is through the limitations on experimental knowledge: (1) the finite number of repetitions of any real experiment necessitates a statistical inference from the finite sample to the infinite population which contains the theoretical meaning of the experiment; (2) the finite value of the action quantum h, which makes it impossible to observe any system without disturbing it finitely; (3) the finite value of the thermal equipartition energy kT, which introduces random noise into almost every physical measurement; (4) the "bluntness" of mechanical and optical probes. The effects of these limitations on the cost and duration of experiments can be reduced greatly by mathematical techniques which are well known in various other fields of science, but which are relatively little used by physicists.

11. Confirmation of polymer and dimer absorptions in the ultraviolet spectra of alcohols. GLADYS A. ANSLOW, IRENE S. WHITE, AND RIKA SARFATY, *Smith College.*

12. Highfrequency electromechanical filters. C. R. MINGINS, S. BARTNOFF, AND L. A. HOWARD, *Tufts College.*

13. The equivalent electrical network of a quartz plate used as a filter. A. D. FROST, C. R. MINGINS, AND R. W. PERRY, *Tufts College.*

14. Stepped-surface piezoelectric filters. R. R. MCDONOUGH, D. W. MACLEOD, AND G. A. LARSON, *Tufts College.*

During the day the Research Laboratories of the Tufts College Physics Department were open for inspection. Work in progress included research in cosmic-rays, the upper atmosphere, piezoelectric crystals, and optical crystals.

A. G. HILL, *Secretary*

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Proceedings of the American Association of Physics Teachers

The New York Meeting, January 28–February 2, 1952

THE twenty-first annual meeting of the American Association of Physics Teachers was held January 28–February 2, 1952, at Barnard College and Columbia University.

The AAPT sessions in Milbank Hall of Barnard College were well attended in spite of the fact that many of the members divided their time between these sessions and those of the American Physical Society. A good deal of the time there were six simultaneous sessions of the APS in the various lecture halls of Columbia, across the street from Barnard.

In additon to the usual run of invited and contributed papers, we had a most interesting symposium of the Oak Ridge Institute of Nuclear Studies with glimpses into current research problems and suggestions for incorporating pile theory into advanced undergraduate physics. There was also a stimulating symposium on physics teaching with renewed consideration of the role of physics in general education and of the virtues of the historical approach and of the "block-and-gap" presentation.

The joint session with the American Physical Society on Friday afternoon found McMillin Academic Theater packed to the rafters. Following the address of DR. LAURITSEN as retiring president of the APS, PRESIDENT MARK ZEMANSKY introduced DR. DUANE ROLLER as chairman of the AAPT Committee on Awards. Dr. Roller presented the Oersted Medal to PROFESSOR A. A. KNOWLTON, now retired after many years at Reed College, and Professor Knowlton told us, from his long experience, about "Opportunities and Rewards in Physics Teaching." Then Dr. Zemansky introduced Nobel Laureate ENRICO FERMI as the tenth Richtmyer Memorial lecturer. Professor Fermi gave us a most interesting story of the newer particles of nuclear physics, pi-mesons, mu-mesons, etc., and of the use of the high energy proton and deuteron beams of the Chicago cyclotron in studying their properties.

The joint banquet at the Hotel New Yorker on Friday evening was also well attended by members of both societies, with the two presidents sharing

the honors. President Zemansky introduced DR. PEIERLS of Birmingham, England, with a little tale about how his small son called him a "dope" because he claimed competence in only two or three fields of physics whereas the boy understood that Einstein is a master of six fields. Before going into his talk about the current situation of physicists in England, Dr. Peierls gracefully explained that his own son has much the same opinion about him. The dinner was a pleasant occasion for renewing old friendships—as were the luncheons of the AAPT at Barnard and the encounters in the corridors between and during sessions.

The business session on Saturday morning went off quickly and uneventfully with appropriate expressions of thanks to the people and institutions who had cooperated to make this a successful meeting and with good wishes for the new president, PROFESSOR W. S. WEBB, as he assumed his new office and exercised his new authority by adjourning the meeting.

HUGH C. WOLFE
The Cooper Union School of Engineering

Invited Papers and Reports

Papers and Special Events

Demonstration of short film-loops. JOHN J. HEILEMANN, *Ursinus College.*

University laboratories utilize enriched stable isotopes. C. P. KEIM, *Oak Ridge National Laboratory.*—Since electromagnetically enriched stable isotopes became available in 1946, 37 university research laboratories have received nearly 500 shipments of 140 different isotopes, representing approximately 35 elements. At least 125 reports have been indexed from the literature, particularly from *The Physical Review*, such research having been accomplished with these enriched isotopes. Some of the fields being investigated are nuclear energy levels, investigations of radioactive isotopes produced from stable isotopes, hyperfine spectroscopy, isotope-shift effects, nuclear spin, nuclear magnetic moments, beta-spectra, isomerism, photoneutron and photoproton emission, cross section studies for neutron and charged particle bombardments, superconductivity measurements, and photographic emulsion applications of an enriched isotope for neutron detection.

Symposium of Oak Ridge Institute of Nuclear Studies

Nuclear reactors as research implements. A. H. SNELL, *Oak Ridge National Laboratory.*

Present status of nuclear reactor theory. A. M. WEINBERG, *Oak Ridge National Laboratory.*

Nuclear research with Van de Graaff accelerators. T. W. BONNER, *The Rice Institute.*

The significance of complementarity for life sciences. W. G. POLLARD, *Oak Ridge Institute of Nuclear Studies.*

Joint Session with the American Physical Society

Some investigations of light nuclei—address of the retiring President of the American Physical Society. CHARLES C. LAURITSEN, *California Institute of Technology.*

Oersted Medal Ceremony

Presentation of the Oersted Medal of the American Association of Physics Teachers to Professor Ansel Alphonso Knowlton. MARK W. ZEMANSKY, *President of the Association*, and DUANE ROLLER, *Chairman of the Committee on Awards.*

Opportunities and rewards in physics teaching. A. A. KNOWLTON, retired, *Reed College.*

Tenth Richtmyer Memorial Lecture of the Association

Large cyclotron research at the University of Chicago. ENRICO FERMI, *University of Chicago.*

Symposium: Methodology in the Teaching of Physics

What general education does for the student. ERIC M. ROGERS, *Princeton University.*

Methodology in physical science. GERALD J. HOLTON, *Harvard University.*

The use of historical material in teaching science at the college level. JAMES B. CONANT, *Harvard University.*

Panel Discussion: Activities and Objectives of Regional Sections. Presiding: WILL V. NORRIS, *University of Oregon.*

Representatives of the various sections of the Association presented reviews of their present programs, plans for growth, and suggestions for greater service of the AAPT to its membership. The chairman indicated that all present should feel free to enter into a frank discussion of these reports. After Secretary Robert Paton of the AAPT gave a brief history of the background for these discussions, the chairman called upon the representatives in order of the seniority of the sections which they represented. Dr. Paton emphasized that the officers of our Association were always eager to help the membership, and stated that he would be glad to channel to appropriate officers any concrete ideas.

Organization, objectives, and activities of the AAPT Western Pennsylvania Section. BERNARD L. BRINKER, *St. Vincent College.*—A review of the work of the Section, and of the Pennsylvania Conference of College Physics Teachers was presented. Problems currently under consideration are (a) the recommendation of minimum requirements for the certification of physics teachers in the secondary schools of Pennsylvania; (b) the stimulation of renewed interest in the AAPT Cooperative Tests for College Students; (c) the operation of a student section

of the American Institute of Physics, and an associated seminar for physics majors; (d) science conferences for students.

Activities in Kentucky. D. M. BENNETT, *University of Louisville*, reporting for R. A. Loring.—The Kentucky Section has three regular meetings a year. The fall meeting is held at the University of Kentucky in Lexington. This is the research meeting at which members report on progress, and graduate students may present papers. The winter meeting is a roving meeting, being held at Western Kentucky State College this year. At this meeting mutual problems are discussed. The spring meeting is held at the University of Louisville in conjunction with the Kentucky Education Association, and at this meeting mutual problems of colleges and secondary schools are discussed. Teachers of physics in high schools are urged to give papers and take part in the discussions. An important feature of each meeting is the luncheon, at which much informal exchange of ideas takes place.

A Secretary reports. WILL V. NORRIS, *University of Oregon*.—Three types of topics are included in programs of the Oregon Section: (a) reviews of current advances in physics; (b) reports of researches in progress at member institutions; (c) improvements found successful in teaching. The preparation of young people in high schools for subsequent more advanced study of physics has occupied a considerable portion of discussion time, and it has been recognized that the major obstacles to a proper coordination of the efforts of teachers in high schools and in colleges are the disparities in facilities, in teaching responsibilities, and in class sizes in the two types of educational institutions.

High school senior science scholarships. C. J. OVERBECK, *Northwestern University*.—The Chicago Section might well follow the example of the Southern California Section in making widely known the need of science scholarships for high school seniors who plan to enter college and in discovering appropriate sources of funds for maintaining such scholarships. Indeed, all regional Sections of the AAPT might take this project as one of their major activities.

A brief analysis was given of the reasons why some sections continue to work enthusiastically, while others appear to be in need of stimulation. The latter condition usually follows a series of unattractive programs or a lack of some goal that is a continual inspiration. The value of cooperation among members of a section in work on long-term projects was emphasized.

Activities of the Chesapeake Section. E. R. PINKSTON, *U. S. Naval Academy*.—In the area represented by the Chesapeake Section—the District of Columbia, Maryland, Virginia, and Delaware—there are approximately 300 Association members. The primary objective of the Section is the advancement of the teaching of physics. Its usual activities consist of an annual meeting in the spring with invited speakers and contributed papers, and an evening meeting in the fall. Last spring the meeting was held at the University of Delaware and the invited speakers were Professor Mark Zemansky, City College of New York, Professor C. W. Ufford, University of Pennsylvania, and Professor L. C. Green, Haverford College. For the evening meeting last fall, there was a panel discussion concerned with the possibility of mutual aid between secondary school and university physics teachers.

Activities in the Rocky Mountains. BYRON E. COHEN, *University of Denver*.—In the Colorado-Wyoming Section research interest is increasing. A new physics building is nearing completion at the University of Colorado; a new high altitude laboratory unit was completed at Echo Lake during the summer of 1951. Three parties of physicists had departed for the Near East to make observations at the time of the total solar eclipse of February 25, 1952. These are University of Colorado, under W. Pietenpol, Zodiacial Light; University of Denver, Byron Cohen, Daylight Luminescence; and Harvard-Colorado Observatory, John Evans, Corona Measurements.

Two major emergencies were reported: flood and fire damage at Colorado State A and M College; fire damage at the University of Denver.

Concern was expressed over the present physics courses for the nonscience majors. It was believed that these courses should be of greater value and interest to able students. The Colorado-Wyoming Section encourages interest in science by acting as sponsor of the Junior Science Academy in that area.

Exchange of information. O. L. RAILSBACK, *University of Illinois*.—The Illinois Section cooperated with the University of Illinois in arranging two four-day physics programs in two consecutive summers in order to bring high school teachers up to date in modern physics, the work being largely in particle physics. About 25 participated. It was suggested that various members of AAPT prepare comprehensive survey lectures in some area of physics and be willing to give them to different groups in several sections of their states. The AAPT would perhaps sponsor such a project.

Local opinion seems to desire that Section representation to the Council of AAPT be made more significant, and that useful ideas of teaching methods, new devices, etc., should be written up and sent around to all sections for their use among local members.

Suggestions from Eastern Pennsylvania. C. A. HODGES, *Temple University*.—Local sections of the AAPT might well cooperate with their state science councils in giving tests to prospective college students in order to encourage and obtain good prospects for physics. This project could be aided by scholarships from various colleges in the area. The AAPT could well arrange for summer employment in industry for students who are interested in physics.

Crusading points for the AAPT. WILLARD GEER, *University of Southern California*.—The seriousness of the shortage of scientists and engineers is sufficient to engage

the attention of all members of the AAPT. A point of strangulation is seen at the precollege level where many large high schools have enrollments of less than one percent in trigonometry. Factors contributing to the neglect of this and other key high school studies were discussed and courses of action for all members suggested.

Essential crusading points for AAPT are as follows: (1) the need for long range plans to recognize the desire and aims of all physics teachers to turn out top grade physics students; (2) the desirability that subject matter be driven down into the earlier years of the curriculum, instead of always advancing it to later years, as by leaving trigonometry for college, and thus delaying the real engineering student; (3) the AAPT should take a strong stand against the forces which are preventing high school students from becoming scientists. It is a fact that only 18 students out of 1800 complete trigonometry in a high-I. Q. high school in Los Angeles.

Reaction of a new section. R. R. PALMER, *Beloit College.*—The Wisconsin Section is young—it has been a Section of the AAPT for only four years. Its activities have been centered around an annual meeting at which time problems of mutual interest are discussed. The Section desires to expand its activities. In particular, members have expressed concern for the problems of the high school physics teacher. It is hoped that ideas from this meeting will serve to suggest ways of expanding its activities in this direction.

Two years ago an innovation was introduced which met with considerable success. The time scheduled for the meeting of the Section was increased from one day to a day and a half to allow time to schedule visits to two local industries. This part of the program was so favorably received that the plan is to be continued. Since the Section meets at a different college each year, the membership will become better acquainted with the various industries in the state.

What universities can do to help the secondary school physics teacher. C. M. LOYD, *West Virginia Institute of Technology.*—The Appalachian Section canvassed thirty institutions in the eastern part of the U. S. and deplored the fact that twenty-eight would not grant master's degrees in physics based on summer work alone and that two would do so only under certain special conditions. Many high school teachers had indicated a real interest in a summer graduate program, if available.

It was suggested that an appeal be made to the universities to set up well-publicized cyclic programs for the up-grading of high school teachers of physics. It was pointed out that a good many of these people would need to begin work at the junior level and some would like to carry it through to the master's degree level.

DR. PAUL KIRKPATRICK was asked why there was no section of the AAPT in the San Francisco area and he replied by letter to the chairman in part as follows: "It seems to me that we have a sufficient concentration of AAPT members around here to operate a section, but so far as I know there has never been any agitation to organize one. I think people in regions of scientific concentration, where they have an abundance of scientific activity and human contact, are less likely to feel the need of more organization than are those in less dense districts."

After each of the above talks, the members present entered into discussion of the various points raised, the general feeling being that this presentation of the problems of sections would prove helpful to all representatives.

Contributed Papers, with Abstracts

1. The graduate school origins of American physicists. JOHN N. COOPER, *Ohio State University.*—There are approximately 3000 physicists with a Ph.D. degree listed in the 1949 edition of *American Men of Science.* The numbers of these degrees granted by each of the major graduate schools have been tabulated. More than 1800 of the degrees were awarded by the thirteen leading institutions, each of which granted over 100 Ph.D. degrees. The University of Chicago has awarded the largest, 230. Following Chicago are Cornell, California (Berkeley), California Institute of Technology, Michigan, Johns Hopkins, Harvard, Columbia, Yale, Wisconsin, Massachusetts Institute of Technology, Princeton, and Illinois in that order. Some 60 other North American schools and 50 European institutions have granted one or more of the Ph.D. degrees.

A survey of the degrees granted during seven five-year periods reveals that significant changes in the list of most productive institutions have occurred. Particularly outstanding has been the rise of California and California Institute of Technology.

2. Let's be more specific. DONALD M. BENNETT, *University of Louisville.*—In most textbooks on physics, the author starts from fundamental laws, and by the application of mathematics to a line of reasoning derives a formula suitable to the case at hand. The student, in his study of this particular problem, will substitute numbers for letters in the formula, and believe he has accomplished something. In an attempt to induce the same thought process in the student as that followed by the author, the student is asked to derive the equation, which in most cases means a temporary memorization. A more satisfactory method has been found in requiring the student to work each individual problem from basic principles. In effect then, he must derive his formula many times in specific cases. This repetition tends to fix principles and increase familiarity with the mathematical manipulations required. The transition to the abstract then becomes a solution of an already solved problem using letters rather than numbers.

3. Where and how much physics should be included in the engineering curriculum. G. P. BREWINGTON, *Lawrence Institute of Technology.*—This first year after the midyear of the present century has been set aside by the American Society for Engineering Education as a year of evaluation. Since freshman and sophomore physics courses have been traditionally a part of the engineering curriculum, it is to be expected that many engineering educators will look

rather critically at physics courses. These men will seek to determine what the objectives of physics are and how well they coordinate with those other departments. It is suggested that physics teachers become equally active in an evaluation of their courses. The largest single customer for the product of the average physics department is the engineering college. We should therefore make a "customer-product" study. We should face the fact that we can teach little nucleonics, theory of the solid state, etc., in our sophomore courses. Yet for every engineer these topics are rapidly becoming a necessity. Many of the preliminary suggestions include a statement that "an adjusted amount of physics instruction be scheduled in the late sophomore, junior, and senior years." Several alternatives will be discussed.

4. "Integration" as an objective of physical science courses. IAN G. BARBOUR, *Kalamazoo College.*—Along with objectives involving the content and methods of science and specific student abilities such as logical thought, "integration" is often mentioned as one objective of physical science courses in general education for non-science majors. This integration may involve (a) the unity and interrelations within one or several scientific disciplines and (b) consideration of the relation of science to other areas of life (to society and history, to man's thinking, etc.). It may stress relationships at the practical level (e.g., technology) or in the realm of method and interaction of ideas. It may be dealt with explicitly and separately, or implicitly throughout the course. There was also discussed the contribution to these various aspects of "integration" as achieved through various types of approach, such as the Harvard case histories, the "problem-unit" approach, conventional or "block-and-gap" presentation, etc.

5. Simple models for the quantum-mechanical, many-body problems. R. J. HARRISON, *Battelle Memorial Institute.*—An understanding of the rather abstract concepts involved in the quantum mechanics of a many-body system is facilitated by considering simple problems which may be solved easily and completely. The two examples described are the one-dimensional equivalents of the ideal gas and the "hard sphere" gas, respectively. Both demonstrate the characteristic feature of the many-body problem in quantum mechanics; namely, that their wave functions do not merely represent a combination of independent one-particle probabilities, but include correlations between positions of the different particles. One can show how these correlations for the ideal gas model depend strongly on the symmetry of the wave function, and then discuss changes introduced by a repulsive potential. In the linear hard-sphere model, one is led to the quantum-mechanical analog of the "effective volume" concept of classical theory. The zero-point energy in this model is independent of statistics. (Experiments on liquid He^3 and He^4 suggest that this may be approximately true in three dimensions.) These simple examples may be used to clarify many features of the more complex many-body problems that one meets in atomic, molecular, and solid state theory.

6. Vector division in introductory physics. J. G. WINANS, *University of Wisconsin.*—Physical quantities based on length, time, and force (two vectors and one scalar) are defined by division as well as by multiplication of vectors. Pressure defined as force/area, mass as force/acceleration, longitudinal force constant as force/displacement all represent division of parallel vectors. These quantities are scalar with no direction. Small angle $d\theta = ds/r$, surface tension S = force/length, shearing stress = force/area all represent division of perpendicular vectors and the quantities are vectors in a direction perpendicular to both vectors in the ratio. Perhaps the most widely used division of arbitrary vectors is cosine θ, defined as adjacent-side/hypotenuse. The cosine of an angle is a quaternion and can be expressed as a scalar plus a vector. The product of two arbitrary vectors is also a quaternion[1] and is equal to the scalar product plus the vector product. Work is the scalar product, and torque the vector product of the quaternion Fs. A scalar (s) divided by a vector (b) gives a unique vector of magnitude $s/|b|$ in the direction of b because of the quaternion character of cosine θ.

[1] P. Kelland and P. G. Tait, *Introduction to Quaternions* (1882), p. 37.

7. Demonstration experiments to provide data for student computation in electricity. T. J. BLISARD AND B. A. GREENBAUM, *Newark College of Engineering.*—Circuits were set up on large panels of plywood painted yellow to increase the visibility. "Snap" connectors were used where changes in connections and insertion of meters were necessary. Wide scale meters were used. Students could not read the meters from a distance with sufficient precision, but they could see the needle movement while selected students read the meters and placed the values on the blackboard.

(a) *Ohm's law demonstration.* The circuit was attached to the panel permanently; meters and resistor blocks were hooked into conveniently placed holes. The resistor blocks consisted of radio resistors mounted on wooden blocks and connected to snap terminals. The resistance of each resistor was measured with a Leeds and Northrup Bridge and the measured and rated values were printed on the block. The board was set up with two groups of three parallel resistors in series plus a single resistor in series. Series, parallel, and series-parallel combinations could be used. Comparison of experimental with measured values of resistance indicated that results were consistent with Ohm's Law.

(b) *Energy loss in power transmission.* The apparatus consisted of a variac, ammeter, voltmeter, variable load (lamp-bank), and a simulated line resistance (coil of approximately 2 ohms). All of these items except the variac were mounted on a panel. The procedure followed was to maintain the load at a constant 50 volts and gradually increase the current through it. This was accomplished by adjusting both the variac and the variable load. At each setting the voltage across the line at the input was also recorded. Thus, the power loss in the line was the difference between $V \times I$ (watt) at the input (variac) and $V \times I$ (watt) at the load. A second set of data was

obtained in which the constant voltage at the load was kept at 100 volts. The observations clearly indicated the advantage of transmitting power at a high voltage.

8. A demonstration of the conservation of energy for student computation. T. J. BLISARD AND C. H. DUURSEMA, *Newark College of Engineering.*—Problems are meaningful to students if they develop from a real situation. To provide such situations, the staff of the physics department at Newark College of Engineering revised its laboratory program to include situations providing practice in sustained computation similar to that encountered by the practicing engineer. The data for the computations are taken from demonstrations set up and operated in the classroom. Students gain understanding, through using these "live data," superior to the results obtained in solving book problems. The authors devised an experiment which involves the determination of the potential and kinetic energies of a moving mass at various positions in its path and illustrates the conservation of energy throughout the motion. The student, by applying basic physical principles and correct mathematical analysis, may obtain results of acceptable precision. The curves which he graphs present him with visual evidence of the principle of conservation of energy. He finds also that the systematic recording and handling of data reduce the labor involved in sustained computation and result in fewer errors.

9. Improved centripetal force device. OSWALD BLACKWOOD, *University of Pittsburgh.*—In a well-known centripetal force experiment, the revolving body is supported by a bifilar suspension at the end of a hand-driven horizontal, revolving arm. A spiral spring pulls the body toward the center of its circular path. This device has been improved by providing a sensitive radius-indicator similar to that in the Cenco Centripetal Force apparatus. More important, oscillations of the spring are damped out by silicone "bouncing putty." These improvements decrease the difficulty of manipulating the device and increase the accuracy of the determinations.

10. The actinoscope, a device to demonstrate the presence of x-rays. EDWIN P. HEINRICH, *American University.*—There is a need for a cheap, self-contained, rugged, and dramatic device which can be used in demonstration lectures to show the presence of x-rays. For its specific use it does not necessarily have to give a quantitative response. An instrument, the actinoscope, has been devised to meet this need. It makes use of the property of gas discharge tubes that they fire at a lower threshold potential under the influence of ionizing radiation.

Two neon tubes are connected in parallel to a resistance-capacity circuit. While the potential across the capacitor increases, both tubes remain dark. Since the tubes, although of the same type, are unlikely to be exactly identical, one will fire at a lower threshold than the other.

The tube that possesses the slightly higher firing potential is used as a detector of x-rays. Its threshold voltage will be lowered by ionizing radiation. It is necessary, for this instrument, to select two tubes which have threshold potentials very close together. Such a matching of characteristics is not difficult.

Never delude yourself into thinking that you are too overworked by your "professional" subject to have time for reading. There is ample time if you will only use it. It is possible, however, that the real excitement of exploration will not start till after you have left the university. Most undergraduates experience a kind of secondary delayed growth in this respect.

The process of your reading will be cumulative, but your understanding of it will increase with a sudden rapidity which I cannot explain. There seems to be a point in time—with English students it often seems to come after about eighteen to twenty months' work—when the whole pattern starts to fit together, to make sense: and men say to me: "If only I had another two years I should begin to know something about literature." That, of course, is an illusion; but the pieces do start to come together, suddenly, in the magnetic field. One poem, one usage of language, certifieth *another. Words are living things: full of references, overtones, shades of meaning, drawn from every context in which they have been used. Every past usage that you know is liable to bring an enrichment, a decoration, an amplification of meaning to what you read. You cannot know them all.*—T. R. HENN, The Apple and The Spectroscope (Methuen, 1951) by permission.